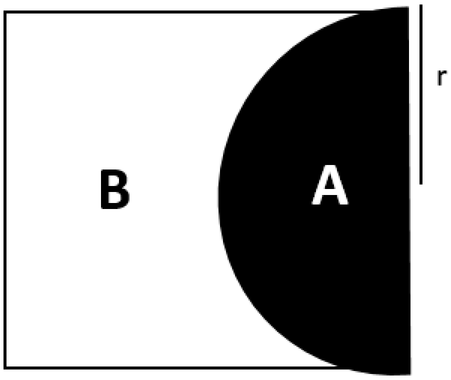
ENGAA Specimen Section 1

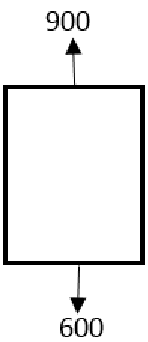
Model Solutions

1



$$\text{Total Area} = (2r)^2 = 4r^2$$
$$\text{Area A} = \frac{1}{2}\pi r^2$$
$$\text{Area B} = 100$$
$$4r^2 - \frac{\pi}{2}r^2 = 100$$
$$r^2 = \frac{100}{4-\frac{\pi}{2}}$$
$$r = \sqrt{\frac{200}{8-\pi}}$$
$$= 10\sqrt{\frac{2}{8-\pi}}$$
$$\therefore \text{One Side} = 2r = 20\sqrt{\frac{2}{8-\pi}}$$

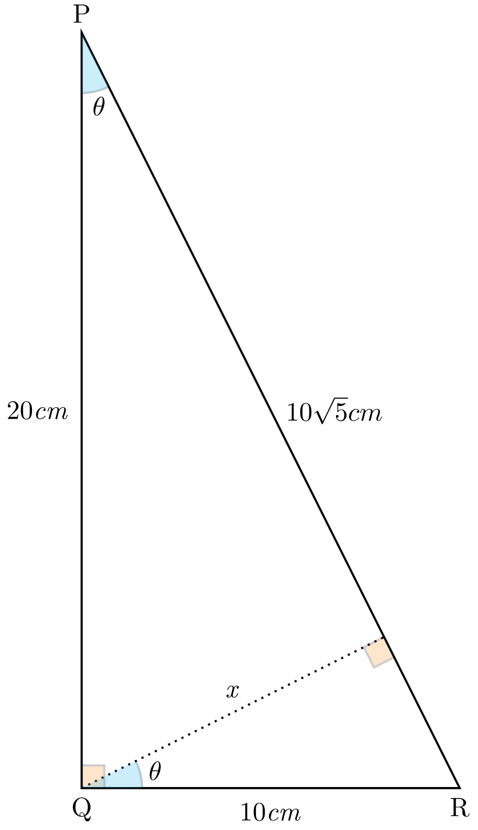
2



Resolving vertically upwards:

$$\text{Resultant Force} = 900 - 600 = 300N$$
$$F = ma$$
$$300 = 60a$$
$$\therefore a = 5ms^{-2} \text{ acting upwards}$$

3



$$RQ : PQ = 10 : 20$$
$$PR = \sqrt{20^2 + 10^2} = 10\sqrt{5}$$

$$sin\theta = \frac{10}{10\sqrt{5}} = \frac{\sqrt{5}}{5}$$
$$x = 20sin\theta = 4\sqrt{5}$$

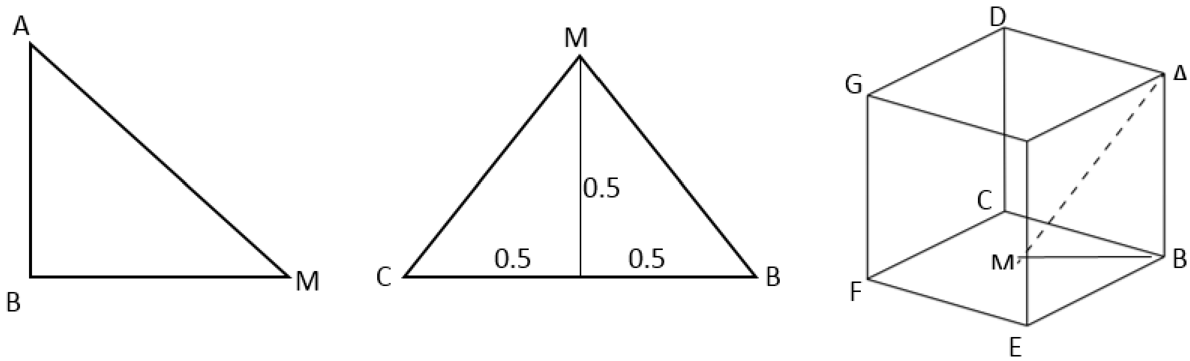
Alternatively, using similar triangles:

$$\frac{x}{10} = \frac{20}{10\sqrt{5}}$$
$$= 4\sqrt{5}$$

4

$$f = \frac{1}{T} = \frac{1}{2}$$
$$v = f\lambda$$
$$= \frac{1}{2} \times 1.5 = 0.75cms^{-1}$$

5



Using Pythagoras' theorem:

$$BM = \sqrt{0.5^2 + 0.5^2} = \frac{\sqrt{2}}{2}$$

$$AM = \sqrt{1^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = \sqrt{\frac{3}{2}}$$

6

$$mv = 30$$

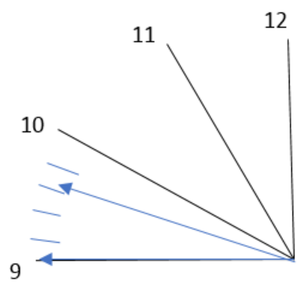
$$\frac{1}{2}mv^2 = 150$$

$$\frac{1}{2}m^2v^2 = 150m$$

$$(30)^2 = 300m$$

$$m = 3kg$$

7



$$\text{Angle between 9:00 and 10:00 } = 30^o$$

$$\text{45 minutes is } \frac{3}{4} \text{ of an hour}$$

$$\frac{3}{4} \times 30^o = 22.5^o$$

8

a) V=IR not $\frac{I}{R}$

b) V $= \frac{W}{Q}$ not $\frac{Q}{W}$

c) Power is measured in JS^{-1}

d) Electric Field strength is measured in NC^{-1}

e) P = IV hence V $= \frac{P}{I}$ (watts per amp)

9

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2}(4+\sqrt{2})(2-\sqrt{2}) \\ &= \frac{1}{2}(8+2\sqrt{2}-4\sqrt{2}-2) \\ &= 3-\sqrt{2} \end{aligned}$$

10

In 24 hours X will undergo 5 half-lives, Y will undergo 3 half-lives:

$$\begin{aligned} A_X &= \left(\frac{1}{2}\right)^5 \cdot 320 = 10Bq \\ A_Y &= \left(\frac{1}{2}\right)^3 \cdot 480 = 60Bq \\ \text{Total Activity} &= 60 + 10 = 70Bq \end{aligned}$$

11

$$\begin{aligned} \text{Volume of a cylinder} &= \pi r^2 h \\ \text{As } h = 2r : \text{Volume }_{\text{cylinder}} &= 2\pi r^3 \end{aligned}$$

$$\begin{aligned} \frac{\text{Volume }_{\text{sphere}}}{\text{Volume }_{\text{cylinder}}} &= \frac{\frac{4}{3}\pi r^3}{2\pi r^3} \\ &= \frac{2}{3} \end{aligned}$$

12

$$\begin{aligned} \text{GPE Loss} &= mgh \\ &= 100 \cdot 10 \cdot 100 \\ &= 100,000J \end{aligned}$$

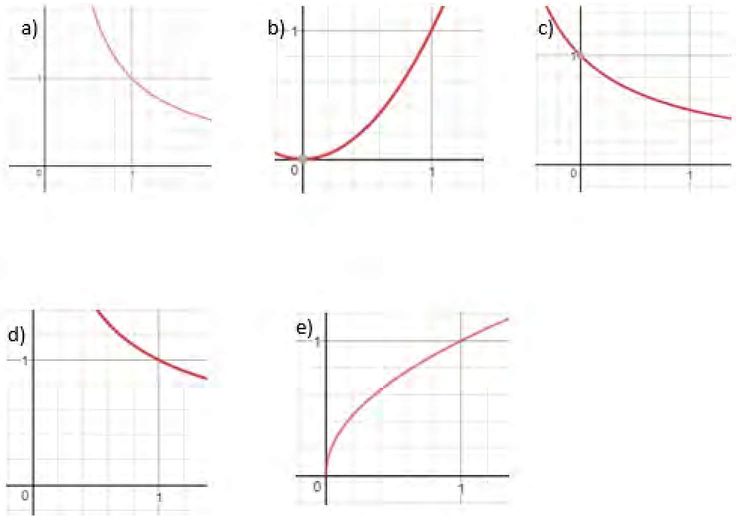
'Constant speed' implies that there is no change in kinetic energy, so all the loss in GPE is lost due resistive forces.

$$10m \text{ along the slope for } 1m \text{ vertical descent } \therefore d = 10 \times 100 = 1000m$$

$$\begin{aligned} \text{Using the Work-Energy Principle: } 100,000J &= 1000m \times F \\ F &= 100N \end{aligned}$$

13

By sketching basic graphs, we can deduce it is not x^2, $\frac{1}{1+x}$ or $\sqrt{x}$ as $f(x) < 1$ in this range for these:



We know $\sqrt{x} > x$ for $0 < x < 1$. Therefore $\frac{1}{x} > \frac{1}{\sqrt{x}}$ for $0 < x < 1$ because the fraction is greater when the value on the denominator is smaller

14

$$v = 0 \qquad\qquad a = -g \qquad\qquad u = 12$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ s &= \frac{0 - 144}{-2 \times 10} \\ &= 7.2m \end{aligned}$$

15

Comparing the similar triangles ABC and ADE:

$$\begin{aligned} \frac{AB}{BC} &= \frac{AD}{DE} \\ \frac{4}{x} &= \frac{(x-4)+4}{x+3} \\ 4x + 12 &= x^2 \\ x^2 - 4x - 12 &= 0 \\ (x-6)(x+2) &= 0 \\ \therefore x &= 6 \text{ because } x \neq -2 \text{ as it represents a length} \\ DE &= x + 3 = 9cm \end{aligned}$$

16

$$\text{Change in KE} = \frac{1}{2}mv^2 \text{ and Change in GPE} = 0$$

$$\text{Work-Energy Principle: } \frac{1}{2}mv^2 = Fd$$

$$d = \frac{mv^2}{2F}$$

17

Inversely proportional implies that as Q increases, P decreases.

$$P \propto \frac{1}{Q^2}$$

$$\therefore \text{If } Q' = 1.4Q : P' = \frac{P}{(1.4)^2}$$

$$= \frac{P}{1.96}$$

$$\frac{1}{1.96} \approx 0.51(2sf)$$

$$\% \text{ change} \approx \frac{1 - 0.51}{1} \times 100$$

$$= 49\%$$

18

In the first decay the proton number decreases by 2, therefore it must be an alpha decay. In the second decay, the nucleon number is unchanged, therefore it must be a beta decay.

Nucleon number: $N \Rightarrow N - 4 \Rightarrow N - 4$ $\therefore P = N - 4$
Proton number: $R \Rightarrow R - 2 \Rightarrow (R - 2) + 1$ $\therefore Q = R - 1$

19

$$x \propto z^2$$

$$z^6 \propto x^3$$

$$y \propto \frac{1}{z^3}$$

$$z^6 \propto \frac{1}{y^2}$$

$$\text{For a constant z: } x^3 \propto \frac{1}{y^2}$$

20

$$\text{Distance travelled} = 10 \times 2 = 20cm$$

$$v = \frac{d}{t}$$

$$t = \frac{0.2}{500}$$

$$= 4 \times 10^{-4}s$$

$$= 0.4ms$$

21

$$\text{Let } QX = 1:$$
$$PM = MR = \frac{1}{2}\left(6 + \frac{3}{2}\right)$$
$$= \frac{15}{4}$$
$$MR = MX + RX$$
$$MX = \frac{15}{4} - \frac{3}{2} = \frac{9}{4}$$
$$\frac{QX}{MX} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$$

22

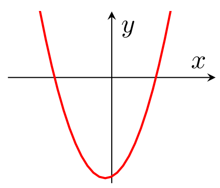
$$\text{P: } a = \frac{10}{24} \neq 2.4ms^{-2}$$
$$\text{Q: } a \approx \frac{58-10}{20} = 2.4ms^{-2}$$

Neither R nor S have $a = 2.4ms^{-2}$ as these have a constant gradient which means velocity is constant and so acceleration is 0. Therefore, Q only is correct.

23

$$x^2 \geq 8 - 2x$$
$$x^2 + 2x - 8 \geq 0$$
$$(x+4)(x-2) \geq 0$$
$$\text{Critical Values: } x = -4, 2$$

This is a positive coefficient quadratic so the graph looks like:



$$\therefore x \geq 2,\ x \leq -4$$

24

a) False; fission involves a larger nucleus being split into two or more smaller nuclei

b) False; half-life is the time taken for the number of undecayed nuclei to halve

c) False, mass number minus atomic number

d) True; a neutron becomes a proton plus an electron in beta decay

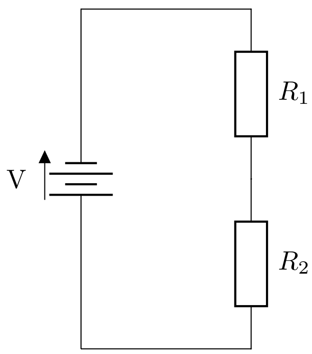
e) False, this describes beta decay not alpha

25

$$\text{Surface area} = 2 \times \pi r^2 + 2\pi rh$$
$$Vol = \pi r^2 h$$

$$\therefore \pi r^2 h = \pi r(2r + 2h)$$
$$rh = 2r + 2h$$
$$rh - 2h = 2r$$
$$h = \frac{2r}{r-2}$$

26



$$P = I_1 V_1$$
$$= \frac{V_1^2}{R_1}$$

$$\text{Potential Divider equation: } V_1 = \frac{R_1}{R_1 + R_2} \times V$$
$$P = \frac{\left(\frac{V_1 R_1}{R_1 + R_2}\right)^2}{R_1}$$
$$= \frac{V^2 R_1^2}{(R_1 + R_2)^2 \times R_1}$$
$$= \frac{V^2 R_1}{(R_1 + R_2)^2}$$

27

After the first rotation, the square's orientation is QRSP. After the reflection in $y = x$ it is QPSR. A reflection in the y-axis will transform the square to its original orientation of PQRS.

28

$$\begin{aligned}
\text{Wavelength} &= unknown \quad \therefore \text{ R false} \\
\text{Speed} &= unknown \quad \therefore \text{ P false} \\
\text{Amplitude} &= \text{half the distance from compression to rarefaction} \\
&= 5mm \div 2 \\
&= 2.5mm \quad \therefore \text{ Q false} \\
\text{Frequency} &= \frac{1}{0.2 \times 10^{-4}s} = 5kHz \quad \therefore \text{ S true only}
\end{aligned}$$

29

$$\begin{aligned}
\frac{x^2-4}{x^2-2x} &= \frac{(x+2)(x-2)}{x(x-2)} \\
&= \frac{x+2}{x}
\end{aligned}$$

30

$$u = 0,\ v = 10, t = 5:$$

$$a = \frac{10}{5} = 2ms^{-2}$$

$$\begin{aligned}
F &= ma \\
3 \times 10^3 - R &= 1000 \times 2 \\
R &= 1000N = 1kN
\end{aligned}$$

31

$$\begin{aligned}
a^x b^{2x} c^{3x} &= 2 \\
\log_{10}(ab^2c^3)^x &= \log_{10} 2 \\
x \log_{10}(ab^2c^3) &= \log_{10} 2 \\
x &= \frac{\log_{10} 2}{\log_{10}(ab^2c^3)}
\end{aligned}$$

32

Taking the initial direction of P as positive, applying the Principle of Conservation of Momentum:

$$\begin{aligned}
u_p m_p - u_q m_q &= v_q m_q - v_p m_p \\
3 \times 2 - r \times 5 &= \frac{r}{2} \times 5 - 1 \times 2 \\
6 + 2 &= \frac{5}{2}r + 5r \\
8 &= \frac{15}{2}r \\
\frac{16}{15} &= r
\end{aligned}$$

33

a) $tan(\frac{3\pi}{4}) = -1$

b) $\log_{10} 100$ simplifies to: $2\log_{10} 10 = 2$

c) $sin^{10}(\frac{\pi}{2}) = 1^{10} = 1$

d) $\log_2 10 > \log_2 8$ and $\log_2 8 = 3$ so $\log_2 10 > 3$

e) $\sqrt{2} \approx 1.41$, therefore $(0.41)^{10} < 1$

$\therefore$ D must be the largest value.

34

Opening the parachute causes air resistance to increase dramatically at first. At terminal velocity, the air resistance is exactly equal to his weight so there is no further acceleration. As his weight remains constant, the air resistance required to balance it and thus reach terminal velocity will be the same before and after release of the parachute. Therefore only A can be correct as it shows the same air resistance for both the flat sections of the graph (where he has reached a terminal velocity).

35

$$\begin{aligned}
(2^x)^2 - 8(2^x) + 15 &= 0 \\
\text{Let } y &= 2^x : \\
y^2 - 8y + 15 &= 0 \\
(y-5)(y-3) &= 0 \\
y &= 5, 3 \\
2^x &= 5, 2^x = 3 \\
x &= \frac{\log_{10} 5}{\log_{10} 2}, x = \frac{\log_{10} 3}{\log_{10} 2} \\
\text{Sum } &= \frac{\log_{10} 5 + \log_{10} 3}{\log_{10} 2} \\
&= \frac{\log_{10} 15}{\log_{10} 2}
\end{aligned}$$

36

- GPE $= mgh$ - The height, h, is not changing so GPE is constant along the horizontal road. Therefore, X cannot be GPE which rules out C and D.
- The mass of the car remains constant so Y cannot be mass, ruling out B and E.
- The velocity of the car increases as the car accelerates. KE $\propto v^2$ so KE increases as the car accelerates. This means KE is not a constant so Z cannot be KE, ruling out A.

By process of elimination F must be correct.

37

$$a \geq b$$
$$-a \leq -b \text{ (multiply by -1 and so flip the inequality sign)}$$
$$\therefore 1 \text{ must be true}$$

$$a^2 + b^2 \geq 2ab$$
$$a^2 - 2ab + b^2 \geq 0$$
$$(a-b)^2 \geq 0 \text{ (this is always true as } x^2 \geq 0 \text{ for all x)}$$
$$\therefore \ 2 \text{ must be true}$$

$$a \geq b$$
$$\text{if } c > 0, \ ac \geq bc$$
$$\text{if } c < 0, \ ac \leq bc$$
$$\therefore 3 \text{ not always true}$$

38

The block is on the point of sliding when the friction takes its *limiting value*: $F = \mu R$. Friction is equal and opposite to P, and therefore the block is stationary, for $0 \leq P \leq \mu R$

When P increases more than μR :

$$P - \mu R = ma$$
$$a = \frac{P - \mu R}{m}$$

For constant μR and m, as P increases, a also increases.

39

$$a_{n+1} = a_n + (-1)^n$$
$$a_1 = 2$$
$$a_2 = 2 + (-1)^1 = 1$$
$$a_3 = 1 + (-1)^2 = 2$$
$$a_4 = 2 + (-1)^3 = 1$$
$$\therefore a_{odd} = 2, \ a_{even} = 1$$

$$\text{Sum of 50 odd} = 50 \times 2 = 100$$
$$\text{Sum of 50 even} = 50 \times 1 = 50$$
$$Total = 150$$

40

Taking the initial direction of motion of the white ball as the positive direction, and applying the Principle of Conservation of Momentum:

$$0.2 \times 3 = 0.2 \times V_R + 0.2 \times 1$$
$$0.4 = 0.2V_R$$
$$V_R = 2$$

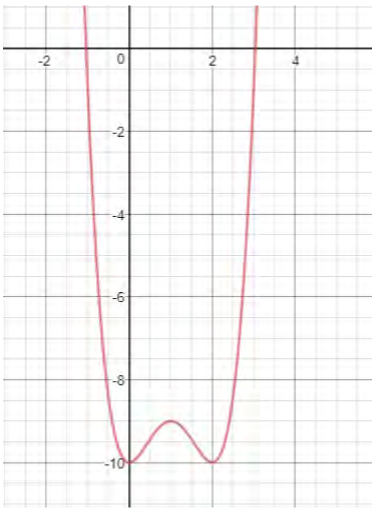
41

$$x^4 - 4x^3 + 4x^2 - 10 = 0$$
$$\frac{dy}{dx} = 4x^3 - 12x^2 + 8x = 0$$
$$x^3 - 3x^2 + 2x = 0$$
$$x(x^2 - 3x + 2) = 0$$
$$x(x-2)(x-1) = 0$$
$\therefore$ 3 turning points at x = 0, 1, 2

Since this is a positive coefficient quartic, we know the order of turning points will be minimum, maximum, then minimum. Substituting the x-values back into the equation:

$$\text{at } x = 0, \ \text{(minimum)}, \ y = -10$$
$$\text{at } x = 1, \ \text{(maximum)}, \ y = -9$$
$$\text{at } x = 2, \ \text{(minimum)}, \ y = -10$$

From the above information we can sketch the quartic as:



So there are 2 roots.

42

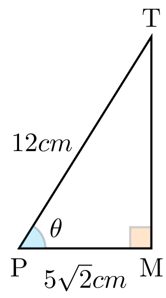
$$a = -g_p, u = 20, v = 0, s = h:$$
$$v^2 = u^2 + 2as$$
$$0 = 20^2 - 2g_ph$$
$$2g_ph = 400$$
$$g_ph = 200$$

A possible pair of values of g_p and h is one whose product is 200. The only such pair is option B, as $5.0 \times 40 = 200$.

43

Let M be the foot of the perpendicular from base PQRS to T, such that TPM is a right-angled triangle. Using Pythagoras' Theorem:

$$PR = \sqrt{10^2 + 10^2}$$
$$= 10\sqrt{2}$$
$$PM = 5\sqrt{2}$$



$$\text{Angle TPM} = cos^{-1}\left(\frac{5\sqrt{2}}{12}\right)$$

44

As the ball falls, all the GPE becomes KE. After the bounce, all the KE becomes GPE as the ball rises:

$$KE = mgh_{max}$$
$$KE \propto h_{max}$$

As KE halves after each bounce, the h_{max} after each bounce is half that of before the bounce:

$$\text{After 1st bounce } h = 8m$$
$$\text{After 2nd Bounce } h = 4m$$
$$\text{After 3rd Bounce } h = 2m$$
$$\text{After 4th Bounce } h = 1m < 1.6m$$

45

A) $6 \log y = x \log a$, therefore not A

B) $\log y = \log a + x \log b$, therefore not B

C) $2 \log y = \log(a + x^b)$, therefore not C

D) $\log y = \log a + b \log x$ which follows the $Y = mX + c$ form of a straight line, where $Y = \log y$ and $X = \log x$, therefore D

E) $x \log y = b \log a$, therefore not E

46

The weight measured on the scale is less than the man's actual weight. This means that the reaction force from the scale on the man is less than his weight. Therefore there is a resultant force on the man acting downwards.

$$F \propto a \text{ (Newton II)}$$

Therefore the direction of acceleration of the man and lift is downwards. The two possible scenarios for this to be the case are: moving upwards with decreasing speed, or moving downwards with increasing speed.

47

For real distinct roots (two) the discriminant of a quadratic must be > 0:

$$ax^2 + (a-2)x - 2 = 0$$
$$b^2 - 4ac > 0$$
$$(a-2)^2 - 4 \cdot a \cdot -2 > 0$$
$$a^2 + 4a + 4 > 0$$
$$(a+2)^2 > 0$$
$$\text{Satisfied for } a \neq 2$$

$\therefore$ The quadratic has two distinct real roots for $a \neq 2$

48

$$u = 12,\ v = 0,\ a = -1.5:$$
$$v^2 = u^2 + 2as$$
$$0 = 12^2 - 2 \cdot 1.5 \cdot s$$
$$s = \frac{144}{3}$$
$$= 48m$$

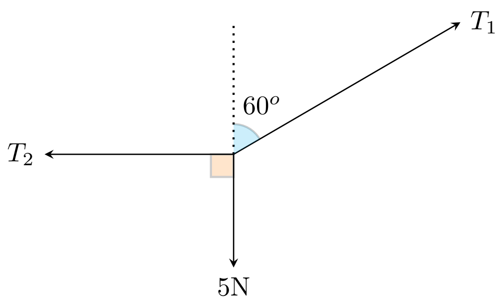
49

The triangle inequality rule states that for a triangle to exist, the sum of the lengths any two sides must be greater than the length of the third side

Given that the three sides must add up to $12cm$, the largest side cannot be longer than $5cm$. If it were $6cm$, then the other two sides would sum $6cm$ thus violating the triangle inequality rule.

$\therefore$ The only possible triangles with integer side lengths are [4,4,4], [5,4,3], and [5,5,2]

50



$$\text{Resolving vertically: } T_1 cos60 = 5N$$
$$T_1 = 10N$$
$$\text{Resolving horizontally: } T_1 sin60 = T_2$$
$$T_2 = 5\sqrt{3}N$$

51

$$PQ^2 = PR^2 + QR^2$$
$$QR = \sqrt{PQ^2 - PR^2}$$
$$PQ = 4 \pm 0.5$$
$$PR = 2 \pm 0.5$$
$$\text{Lower Bound of } PQ^2 - PR^2 = 3.5^2 - 2.5^2$$
$$= (3.5 + 2.5)(3.5 - 2.5)$$
$$= 6 \times 1 = 6$$
$$\therefore \text{Lower bound of QR} = \sqrt{6}$$

52

$$P = \frac{W}{t}$$
$$W = \text{GPE gained}$$
$$= 200h$$
$$\therefore P = \frac{200h}{t}$$

$$\text{Gradient} = \frac{h}{t} = \frac{10-5}{25-15} = 0.5$$
$$P = 200 \times 0.5$$
$$= 100W$$

53

$$-1 \leq tanx \leq 1 \text{ for: } 0 \leq x \leq \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \leq x \leq \pi$$
$$sinx \geq 0.5 \text{ for } \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$$
$$sin2x \geq 0.5 \text{ for } \frac{\pi}{12} \leq x \leq \frac{5\pi}{12}$$
$$\therefore \text{ Interval for which both valid: } \frac{\pi}{12} \leq x \leq \frac{\pi}{4}$$
$$\text{Length} = \frac{\pi}{4} - \frac{\pi}{12}$$
$$= \frac{\pi}{6}$$

54

Resolving forces for the whole system:

$$F = ma$$
$$15000 = 30000a$$
$$a = 0.5ms^{-2}$$

Resolving forces for carriage 2:

$$T = 5000 \times 0.5$$
$$= 2500N$$