ENGAA Specimen
Section 2

Model Solutions

1

1a

The normal contact force (N) acts perpendicular to the surface which rules out B, C and E.
The block is travelling down the slope. Hence friction (μN) acts in the opposite direction to motion; up the slope.

1b

The block has zero acceleration in the direction perpendicular to the slope (otherwise it would be lifting off from the slope or digging into the slope). This means net resultant force when resolved perpendicular to the slope is zero.

1c

Resolve perpendicular to the slope to find N:

$$N - mgcos\alpha = 0$$
$$N = mgcos\alpha$$

Resolve parallel to the slope to find acceleration:

$$mgsin\alpha - \mu N = ma$$
$$\frac{mgsin\alpha - \mu mgcos\alpha}{m} = a$$
$$a = g(sin\alpha - \mu cos\alpha)$$

Given also $u = 0, t = 0, v = v$:

$$v = u + at$$
$$= g(sin\alpha - \mu cos\alpha)t$$

1d

Resolve parallel to the slope to find acceleration:

$$a = \frac{mgsin\alpha - \mu N - kV}{m}$$

$$v = u + at$$
$$V = \frac{mgsin\alpha - \mu mgcos\alpha - kV}{m}t$$

Making V the subject:

$$V = \frac{mg(sin\alpha - \mu cos\alpha)t}{m + kt}$$

Dividing through by t:

$$V = \frac{mg(sin\alpha - \mu cos\alpha)}{\frac{m}{t} + k}$$

as t tends to infinity, $\frac{m}{t}$ tends to zero and V tends to:

$$V = \frac{mg(sin\alpha - \mu cos\alpha)}{k}$$

2

2a

We are working with approximate values, so we can calculate the order of magnitude of the answer once converted into SI units.

$$A = \pi r^2$$
$$A \approx \pi(10^{-2})^2 = 10^{-3}$$

Resistance of each cable is found as:

$$R = \frac{\rho L}{A}$$
$$R \approx \frac{(10^{-8})(10^5)}{10^{-3}} = 1$$

We can deduce the resistance of one cable is:

$$R = 3.7\Omega$$

The resistance per km of cable:

$$R = \frac{3.7\Omega}{100km} = 0.037\Omega km^{-1}$$
$$= 37m\Omega km^{-1}$$

2b

$$P = IV$$
$$\text{Current through cables: } = \frac{1GW}{400kV} = 2500A$$
$$P_{loss} = I^2R$$
$$= 2500^2 \cdot 3.7 \approx 23MW$$
$$\text{Total Power loss in both cables} = 46MW$$

2c

$$\text{Power } [W] = \frac{\text{Energy } [J]}{\text{Time } [s]}$$

Hence 1GW of power (1×10^9W) multiplied by 1 hour of time (3600s) gives $3.6 \times 10^{12} J$ of energy.

2d

- Eliminate D and E: the law of conservation of energy is not violated because the energy is not destroyed, but simply converted into other non-useful forms such as heat.
- Eliminate A: 'Efficiency' is with reference to the amount of energy from the fuel source which is converted into useful electrical energy – this does not refer to the amount of electrical energy which is generated or consumed
- Eliminate B: Here 'efficiency' is with reference to how much of the consumed energy is converted into useful other forms – this is not relevant to the difference between generation and consumption
- By process of elimination C is the correct answer

3

3a

- $v = 0$ when: $t = 0$ and $t = T$ (roots)
- $t = T$ is a repeated root so the graph is tangential to the t-axis at T
- $t = 0$ is not a repeated root, so the graph does not meet the t-axis tangentially at the origin
- Outside $0 < t < T$, the object is at rest, so $v = 0$ for all other values
- For $t > 0$, t is always positive and as $(t - T)^2$ will always be positive, the cubic must be above the x-axis

3b

$$acceleration = \frac{d}{dx}(velocity)$$

$$\begin{aligned}
&\text{Differentiate } v(t) = at(t-T)^2 \text{ using product rule:}\\
A(t) &= a(t-T)^2 + 2at(t-T)\\
&= (t-T)a[t-T+2t]\\
&= a(t-T)(3t-T)
\end{aligned}$$

3c

$$\begin{aligned}
A'(t) &= a[3(t-T) + (3t-T)]\\
&= 0 \text{ (at turning points)}\\
&= 3t - 3T + 3t - T\\
&= 6t - 4T
\end{aligned}$$

$$\text{At t} = \frac{2T}{3}:$$

$$\begin{aligned}
\text{Acceleration} &= a\left(-\frac{1}{3}T\right)\cdot T\\
&= -\frac{1}{3}aT^2
\end{aligned}$$

3d

- On a velocity-time graph, acceleration = gradient
- Steepest positive gradient (on Graph D) is at t=0
- Hence $a_{max} = a \cdot (-T) \cdot (-T) = aT^2$

3e

From $t = T$ to $t = 2T$, velocity is 0, hence there is no further displacement.

$$\begin{aligned}
&a\int_0^T (t^3 - 2Tt^2 + T^2 t)dt\\
&= a\left[\frac{1}{4}T^4 - \frac{2}{3}T^4 + \frac{1}{2}T^4\right]\\
&= \frac{1}{12}aT^4
\end{aligned}$$

4

4a

$$\text{GPE loss} = \text{KE gain}$$
$$MgH = \frac{1}{2}Mv^2$$
$$v = \sqrt{2gH}$$

4b

Total mechanical energy in the system remains constant as no external forces act:

$$ME_b = ME_a = MgH$$
$$Mg(2R) + \frac{1}{2}Mv^2 = MgH$$
$$v^2 = 2g(H - 2R)$$
$$v = \sqrt{2g(H - 2R)}$$

4c

Centripetal acceleration always acts towards the centre of the circle

4d

- If the normal reaction force $R < 0$ the car will lose contact with the track
- In the limiting case, for the car to not lose contact with the track, R will equal 0
- Car accelerates at a constant $\frac{V^2}{R}$ around the circular track
- $F = ma$, hence the centripetal force required to remain in circle $= \frac{MV^2}{R}$

At the top, R = 0:

$$\frac{MV^2}{R} = Mg$$
$$V = \sqrt{gR}$$
$$ME_B = ME_{top}$$
$$MgH = 2MgR + \frac{1}{2}M(gR)$$
$$H = 2R + \frac{1}{2}R$$
$$\therefore H > 2.5R$$