

AQA Physics A-Level

Section 1: Measurements and their Errors Notes

3.1 Measurements and their errors

3.1.1 - Uses of SI units and their prefixes

SI units are the **fundamental** units, they are made up of:

- Mass (m): **kg** (kilograms)
- Length (l): **m** (metres)
- Time (t): **s** (seconds)
- Amount of substance (n): **mol** (moles)
- Temperature (t): **K** (kelvin)
- Electric current (I): **A** (amperes)

The SI units of quantities can be **derived** by their equation, e.g. $F=ma$

For example, to find the SI units of force (F) multiply the units of mass and acceleration $\text{kg} \times \text{m} \text{s}^{-2}$ gives $\text{kgm} \text{s}^{-2}$ (Also known as N)

The SI units of voltage can be found by a series of steps:

- $V = \frac{E}{Q}$ where E is energy and Q is charge, $E = \frac{1}{2} m v^2$ so the SI units for energy is $\text{kg} \text{m}^2 \text{s}^{-2}$ (the units for speed (v) are $\text{m} \text{s}^{-1}$ so squaring these gives $\text{m}^2 \text{s}^{-2}$)
- $Q = It$ (where I is current) so the units for Q are **As** (ampere seconds)
- So $V = \frac{\text{kgm}^2 \text{s}^{-2}}{\text{As}}$ $V = \text{kgm}^2 \text{s}^{-3} \text{A}^{-1}$

Below are the **prefixes** which could be added before any of the above SI units:

Name	Symbol	Multiplier
Tera	T	10^{12}
Giga	G	10^9
Mega	M	10^6
Kilo	k	10^3
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}
Femto	f	10^{-15}

Some examples:

6pF (picofarads) is $6 \times 10^{-12} \text{ F}$

9GΩ (gigaohms) is $9 \times 10^9 \text{ Ω}$

10μm (micrometres) is $10 \times 10^{-6} \text{ m}$

Converting mega electron volts to joules:

$$1\text{eV} = 1.6 \times 10^{-19} \text{ J}$$

e.g. convert 76 MeV to joules:

$$\text{First, convert from MeV to eV by multiplying by } 10^6 \quad 76 \times 10^6 \text{ eV}$$

$$\text{Then convert to joules by multiplying by } 1.6 \times 10^{-19} \quad 1.216 \times 10^{-11} \text{ J}$$

Converting **kWh** (kilowatt hours) to Joules:

$$1 \text{ kW} = 1000 \text{ J/s} \quad 1 \text{ hour} = 3600\text{s}$$

$$1\text{kWh} = 1000 \times 3600$$

$$= 3.6 \times 10^6 \text{ J}$$

$$= 3.6 \text{ MJ}$$

3.1.2 - Limitation of Physical Measurements

Random errors affect **precision**, meaning they cause differences in measurements which causes a spread about the mean. You **cannot** get rid of all random errors.

An example of random error is **electronic noise** in the circuit of an electrical instrument

To reduce random errors:

- Take **at least 3 repeats** and calculate a **mean**, this method also allows **anomalies to be identified**.
- Use **computers/data loggers/cameras** to reduce human error and enable **smaller intervals**.
- Use **appropriate equipment**, e.g a micrometer has higher resolution (0.1 mm) than a ruler (1 mm).

Systematic errors affect **accuracy** and occur due to the apparatus or faults in the experimental method. Systematic errors cause all results to be **too high or too low by the same amount** each time.

An example is a balance that isn't zeroed correctly (**zero error**) or reading a scale at a different angle (this is a **parallax error**).

To reduce systematic error:

- **Calibrate** apparatus by measuring a known value (e.g. weigh 1 kg on a mass balance), if the reading is inaccurate then the systematic error is easily identified.
- In radiation experiments correct for **background radiation** by measuring it beforehand and excluding it from final results.
- Read the **meniscus** (the central curve on the surface of a liquid) **at eye level** (to reduce parallax error) and use **controls** in experiments.

Precision	Precise measurements are consistent, they fluctuate slightly about a mean value - this doesn't indicate the value is accurate
Repeatability	If the original experimenter can redo the experiment with the same equipment and method then get the same results it is repeatable

Reproducibility	If the experiment is redone by a different person or with different techniques and equipment and the same results are found, it is reproducible
Resolution	The smallest change in the quantity being measured that gives a recognisable change in reading
Accuracy	A measurement close to the true value is accurate

The **uncertainty** of a measurement is the bounds in which the accurate value can be expected to lie e.g. $20^{\circ}\text{C} \pm 2^{\circ}\text{C}$, the true value could be within $18\text{-}22^{\circ}\text{C}$

Absolute Uncertainty: uncertainty given as a fixed quantity e.g. $7 \pm 0.6 \text{ V}$

Fractional Uncertainty: uncertainty as a fraction of the measurement e.g. $7 \pm \frac{3}{35} \text{ V}$

Percentage Uncertainty: uncertainty as a percentage of the measurement e.g. $7 \pm 8.6\% \text{ V}$

To reduce percentage and fractional uncertainty, you can measure larger quantities.

Resolution and Uncertainty

Readings are when **one value** is found e.g. reading a thermometer, measurements are when the **difference between 2 readings** is found, e.g. a ruler (as both the starting point and end point are judged).

The **uncertainty in a reading** is **\pm half the smallest division**,
e.g. for a thermometer the smallest division is 1°C so the uncertainty is $\pm 0.5^{\circ}\text{C}$.

The **uncertainty in a measurement** is **at least ± 1 smallest division**,
e.g. a ruler, must include **both** the uncertainty for the start and end value, as each end has $\pm 0.5\text{mm}$, they are added so the uncertainty in the measurement is $\pm 1\text{mm}$.

Digital readings and given values will either have the uncertainty quoted or assumed to be **\pm the last significant digit** e.g. $3.2 \pm 0.1 \text{ V}$, the **resolution** of an instrument affects its uncertainty.

For **repeated data** the uncertainty is **half the range** (largest - smallest value), show as **mean $\pm \frac{\text{range}}{2}$** .

You can reduce uncertainty by **fixing one end** of a ruler as only the uncertainty in **one reading** is included. You can also reduce uncertainty by measuring **multiple instances**,
e.g. to find the time for 1 swing of a pendulum by measuring the time for 10 giving e.g. $6.2 \pm 0.1 \text{ s}$, the time for 1 swing is $0.62 \pm 0.01\text{s}$ (**the uncertainty is also divided by 10**).

Uncertainties should be given to the **same number of significant figures** as the data.

Combining uncertainties

- **Adding / subtracting data - ADD ABSOLUTE UNCERTAINTIES**

E.g. A thermometer with an uncertainty of ± 0.5 K shows the temperature of water falling from 298 ± 0.5 K to 273 ± 0.5 K, what is the difference in temperature?

$$298 - 273 = 25\text{K} \quad 0.5 + 0.5 = 1\text{K (add absolute uncertainties)} \quad \text{difference} = 25 \pm 1 \text{ K}$$

- **Multiplying / dividing data - ADD PERCENTAGE UNCERTAINTIES**

E.g. a force of 91 ± 3 N is applied to a mass of 7 ± 0.2 kg, what is the acceleration of the mass?

$$a = F/m = 91/7 = 13 \text{ m s}^{-2} \quad \text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{value}} \times 100$$

$$\text{Work out \% uncertainties } \frac{3}{91} \times 100 + \frac{0.2}{7} \times 100 = 3.3\% + 2.9\% \quad \text{add \% uncertainties} \\ = 6.2\%$$

So $a = 13 \pm 6.2\% \text{ m s}^{-2}$ 6.2% of 13 is 0.8

$$a = 13 \pm 0.8 \text{ m s}^{-2}$$

- **Raising to a power - MULTIPLY PERCENTAGE UNCERTAINTY BY POWER**

The radius of a circle is 5 ± 0.3 cm, what is the percentage uncertainty in the area of the circle?

$$\text{Area} = \pi \times 5^2 = 78.5 \text{ cm}^2$$

$$\text{Area} = \pi r^2$$

$$\% \text{ uncertainty in radius} = \frac{0.3}{5} \times 100 = 6\% \quad \% \text{ uncertainty in area} = 6 \times 2 \text{ (2 is the power from } r^2 \text{)} \\ = 12\%$$

$$78.5 \pm 12\% \text{ cm}^2$$

Uncertainties and graphs

Uncertainties are shown as **error bars** on graphs,

e.g. if the uncertainty is 5mm then have 5 squares of error bar on either side of the point

A line of best fit on a graph should **go through all error bars (excluding anomalous points)**.

The **uncertainty in a gradient** can be found by lines of best and worst fit, this is especially useful when the gradient represents a value such as the acceleration due to gravity:

- Draw a **steepest and shallowest** line of worst fit, it **must** go through all the error bars.
- Calculate the gradient of the line of best and worst fit, the uncertainty is the **difference between the best and worst gradients**.

$$\text{percentage uncertainty} = \frac{|\text{best gradient} - \text{worst gradient}|}{\text{best gradient}} \times 100\%$$

(modulus lines show it's always positive)

When the best and worst lines have different y intercepts, you can find the **uncertainty in the y-intercept**, which is |best y intercept - worst y intercept|:

$$\text{percentage uncertainty} = \frac{|\text{best y intercept} - \text{worst y intercept}|}{\text{best y intercept}} \times 100\%$$

3.1.3 - Estimation of physical quantities

Orders of magnitude - Powers of ten which describe the size of an object, and which can also be used to compare the sizes of objects.

E.g: The diameter of nuclei have an order of magnitude of around 10^{-14} m.

100 m is two orders of magnitude greater than 1m.

You may be asked to give a value to the **nearest order of magnitude**, here you must simply calculate the value the question is asking you for and give it only as a power of ten

E.g If the diameter of a hydrogen atom is 1.06×10^{-10} m, find the approximate area of the entire atom (assuming it is perfectly spherical), to the nearest order of magnitude.

Find the area using $A = \pi r^2$ $\pi \times (0.53 \times 10^{-10})^2 = 8.82 \times 10^{-21} \text{ m}^2 = 1 \times 10^{-20} \text{ m}^2$ (to 1 s.f)

Therefore the area to the nearest order of magnitude is 10^{-20} m^2 .

Estimation is a skill physicists must use in order to approximate the values of physical quantities, in order to make **comparisons**, or to check if a value they've calculated is **reasonable**.