AQA Physics A-level

Section 12: Turning points in physics

Notes

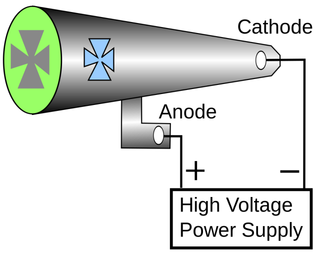
3.12.1 The discovery of the electron

3.12.1.1 - Cathode rays

When a **potential difference is applied across a discharge tube with a low pressure gas** inside of it, the tube will begin to glow with it glowing brightest at the **cathode**. This **glow was called the cathode ray**, and scientists were unsure as to what it was made up of until Thomson showed that cathode rays:

- Have a **mass**, which he measured.
- Have a **negative charge.**
- Have the **same properties** no matter what gas is used in the discharge tube.
- Have a very **large charge to mass ratio**.

Soon after it was concluded that all atoms contained these cathode ray particles, and they were renamed **electrons**.



The process by which the discharge tube begins to glow is outlined below:

1. The high potential difference across the discharge tube will **pull electrons off the gas atoms**, forming **ion and electron pairs**.
2. The **positive gas ions are accelerated towards the cathode** and when they collide with it they release even more electrons.
3. The electrons are **accelerated** along the tube (because the gas is at low pressure, the electrons are accelerated to high speeds) and **collide with gas atoms causing them to become excited**. The atoms will quickly **de-excite and release photons of light**.

The glow is brightest at the cathode because here the **gas ions and electrons can recombine and emit photons of light**.

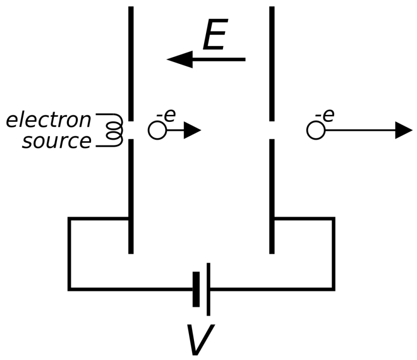
3.12.1.2 - Thermionic emission

Thermionic emission is where a metal is heated until the free electrons on its surface gain enough energy and are emitted.

Electron guns use a potential difference in order to accelerate electrons, which are released from the cathode by heating it **(thermionic emission)**. The electrons are **accelerated towards the anode**, which has a small gap, the electrons which pass through this gap form a narrow electron beam which travels at a constant velocity beyond the anode. The **work done on a charged particle** in an electric field is given by: $\Delta W = Q\Delta V$, therefore the **work done on an electron accelerated through a potential difference V** is:

$$\Delta W = eV$$

Where e is the charge on an electron.



This is usually measured in **electron volts (eV)**, where 1 eV is equal to the **kinetic energy of an electron accelerated across a potential difference of 1 V**.

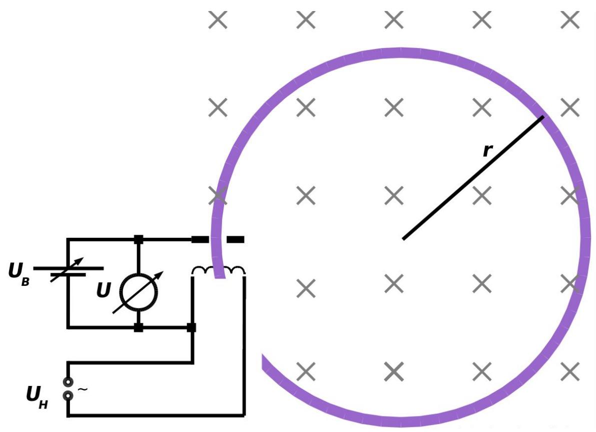
As the electron moves from the cathode towards the anode, its **electrical potential energy is converted into kinetic energy** and so the electron speeds up. Once the electron reaches the anode, its **kinetic energy will be equal to the work done on the electron by the electric field** so:

$$\frac{1}{2}mv^2 = eV$$

3.12.1.3 - Specific charge of the electron

You will need to know how to **determine the specific charge of an electron by any one method**, below are two different methods, however you will only need to learn the one which you feel more comfortable with.

Fine beam tube - this piece of apparatus contains a **low pressure gas** and has a **uniform magnetic field** passing through.



1. Electrons are accelerated using an electron gun and enter the fine beam tube perpendicular to the direction of the field.
2. The magnetic force on the electrons **acts perpendicular to their motion**, and therefore the electrons move in a circular path because the **magnetic force acts as a centripetal force**.
3. As the electrons move through the fine beam tube, they **collide with gas atoms causing them to become excited**, the gas atoms then de-excite releasing photons of light meaning the path of the electrons is visible, so the radius of their circular path can be measured.

$$\frac{m_e v^2}{r} = Bev$$

Can be substituted by $\frac{1}{2}m_e v^2 = eV$

$v^2 = \frac{2eV}{m_e}$ $v = (\frac{2eV}{m_e})^{1/2}$

$$\frac{m_e(\frac{2eV}{m_e})^{1/2}}{r} = Be$$

One of the v's are cancelled while one is replaced using the equation.

$$\frac{m_e 2V}{r^2} = B^2 e$$

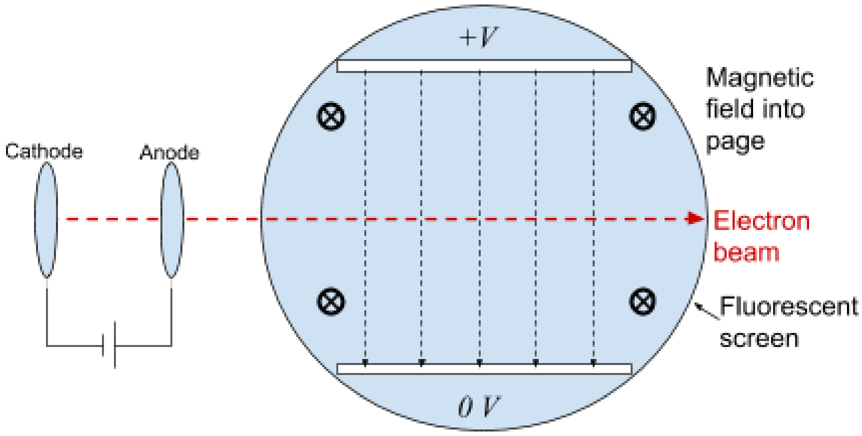
Every term is squared, one e is cancelled from each side, as well as the m_e which is cancelled on the left side.

$$\frac{e}{m_e} = \frac{2V}{B^2 r^2}$$

Rearrange to get the equation for specific charge on the left.

Using the above equation you can find specific charge, as you can measure all the values on the right.

Thomson's crossed fields - this apparatus involves magnetic and electric fields which are perpendicular to each other (as shown in the diagram), where the electric field and magnetic fields deflect the electrons in opposite directions.



1. Electrons are accelerated using an electron gun and enter the apparatus perpendicular to the direction of both fields. As we can see from the apparatus above, the electrons will be deflected upwards by the electric field, while being deflected downwards by the magnetic field (due to Fleming's left hand rule).
2. The **strengths of these fields are adjusted until the electron beam passes through the crossed fields undeflected**, therefore the electric and magnetic forces are equal and opposite.

   Magnetic force = $F = Bev$ Electric force = $F = Ee$ where $E = \frac{V}{d}$, so $F = \frac{Ve}{d}$

$$Bev = \frac{Ve}{d}$$
$$v = \frac{V}{Bd}$$

The kinetic energy of the electron can be given by $\frac{1}{2}m_e v^2 = eV_a$ where V_a is the accelerating voltage, so $v^2 = \frac{2eV_a}{m_e}$.

$$\frac{V^2}{B^2d^2} = \frac{2eV_a}{m_e}$$
$$\frac{e}{m_e} = \frac{V^2}{2B^2d^2V_a}$$

Using the above equation you can find specific charge, as you can measure all the values on the right.

Thomson's determination of specific charge of the electron was significant because it showed that the **specific charge was constant** whatever gas was used to produce the electrons (cathode rays), demonstrating that **all atoms contain electrons**. He went on to propose the plum pudding model of the atom, where electrons are spread uniformly throughout a sphere of positive charge (this was later disproved by Rutherford scattering).

The specific charge of the electron is around 1.76×10^{11} Ckg^{-1}, whereas the specific charge of a hydrogen ion (proton) is 9.58×10^{7} Ckg^{-1}, meaning that the **specific charge of an electron is around 1800 times larger than that of a proton**.

3.12.1.4 - Principle of Millikan's determination of the electronic charge, e

Millikan's oil drop experiment was formed in order to calculate the charge of an electron, below is a diagram of the apparatus used:

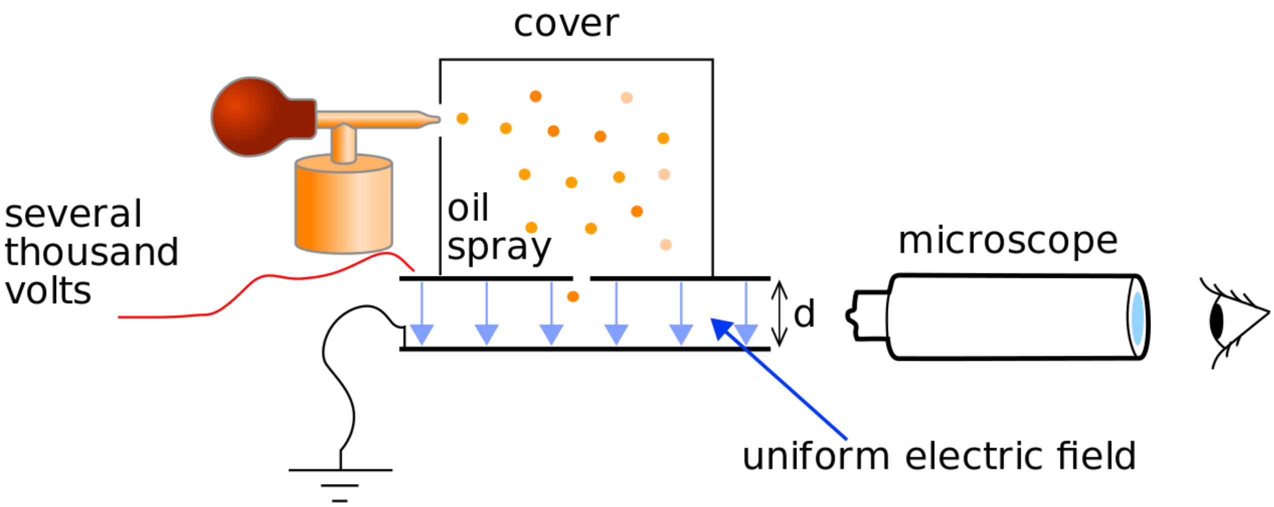


Image source: Gregors, CC BY-SA 3.0

An **atomizer** is used to spray tiny droplets of oil, which are **negatively charged due to friction**. These droplets fall until they reach two parallel plates which form a uniform electric field, as the droplets are charged they will experience an electric force. The strength of the field can be adjusted by changing the potential difference between the plates, until the observed oil droplet becomes **stationary**, meaning that **its weight is equal to the electric force it experiences upwards**:

$$EQ = mg$$

$$\frac{QV}{d} = mg \text{ using } E = \frac{V}{d}$$

Where Q is the charge of the droplet, V is the potential difference across the parallel plates, d is the distance between the plates and m is the mass of the droplet.

From the method above alone we cannot find the magnitude of charge on the oil drop because the **mass of the oil drop is unknown**, therefore the oil drops mass must be measured first. In order to do this, the **potential difference across the plates is removed** so the droplet no longer experiences an electric force upwards, and so will begin to fall. The droplet will experience a resistive force upwards (known as a viscous drag force) and its weight downwards, the **viscous drag force (F)** can be calculated using **Stokes' law**:

$$F = 6\pi\eta rv$$

Where η is the viscosity of the fluid, r is the radius of the object and v is the terminal velocity of the object.

The terminal velocity of the droplet can be measured by using a **microscope with a calibrated graticule and measuring the distance travelled by the droplet in a certain amount of time**. When the droplet is moving at terminal velocity, the **viscous force and weight are equal**:

$$6\pi\eta rv = mg$$

$$mass = volume \times density \Rightarrow m = \tfrac{4}{3}\pi r^3\rho$$

$$6\pi\eta rv = \tfrac{4}{3}\pi r^3\rho g$$

$$r^2 = \frac{9\eta v}{2\rho g}$$

Where ρ is the density of the oil, r is the droplet's radius, and η is the viscosity of the fluid.

Using the above formula, the radius of the oil droplet can be found meaning that its mass can be measured, and so the charge of the droplet can be calculated.

$$\frac{QV}{d} = mg$$

$$\frac{QV}{d} = \tfrac{4}{3}\pi r^3\rho g$$

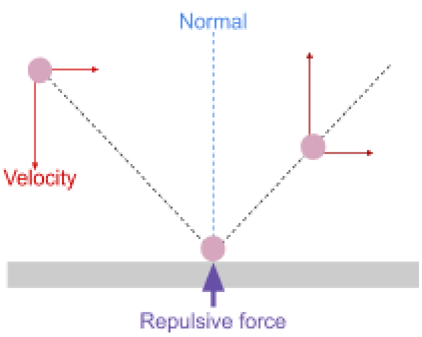
Millikan's results showed that the charge of all the oil droplets he observed was an **integer multiple of 1.6×10^{-19} C**, this is significant because it shows that charge is **quantised**, meaning it exists in **discrete packets** of 1.6×10^{-19} C, which is the smallest possible magnitude of charge - the magnitude of charge carried by an electron.

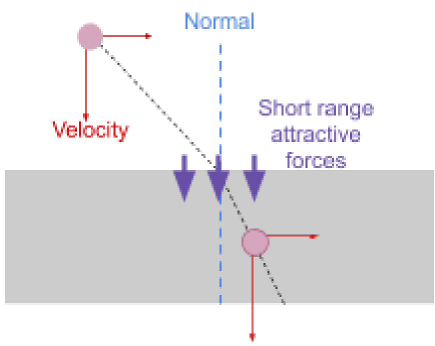
3.12.2 Wave-particle duality

3.12.2.1 - Newton's corpuscular theory of light

Newton theorised that light was formed of **tiny particles called corpuscles**, with the following explanations for these properties of light:

- **Reflection** - the corpuscles collide with the surface and a repulsive force pushes them back, causing their component of **velocity perpendicular to the surface to change direction**, while their component of **velocity parallel to the surface stays the same**.
- **Refraction** - as the corpuscles approach a denser medium, **short-range forces of attraction** cause their component of **velocity perpendicular to the surface to increase**, while the **parallel component of velocity stays the same**, therefore the light will **bend towards the normal**. **According to Newton's explanation light travels faster in denser mediums**.





Even though reflection and refraction can be explained using Newton's theory, the diffraction of light can't be, therefore there were other theories of light such as **Huygens' wave theory**.

Huygen believed that light was a wave and that **every point on a wavefront is a point source to secondary wavelets, which spread out to form the next wavefront (Huygen's principle)**, as shown in the diagram below:

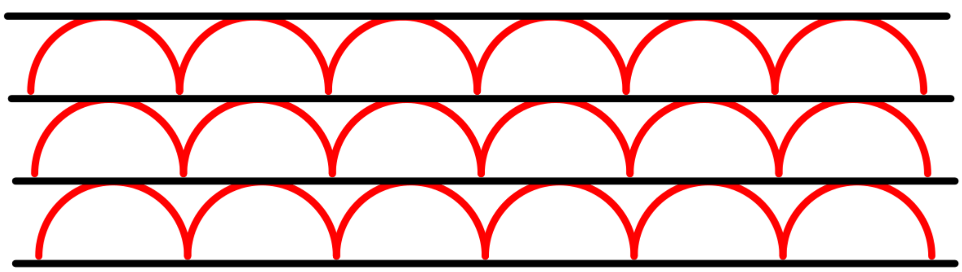
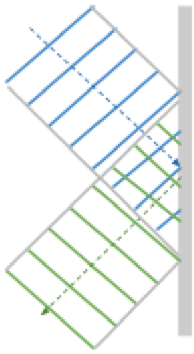


Image source: Tiger66,CC BY-SA 4.0, Image is cropped

Using Huygen's principle, reflection and refraction could be easily explained:

- **Reflection** - as the whole wavefront will not reach the surface at once (unless it is travelling perpendicular to the surface), wavelets spread away from the surface once they reach it and rejoin with others to reform the reflected wavefront.



- **Refraction** - it was **assumed that light travels slower in a more dense medium**, therefore as it entered a more optically dense medium it would slow down and therefore bend towards the normal.

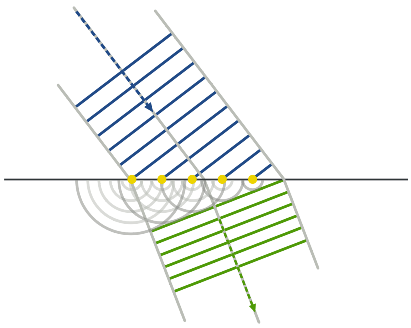
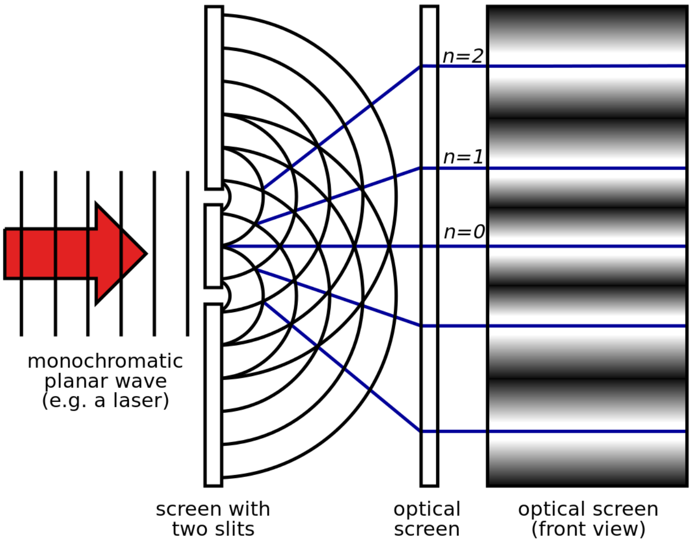


Image source: Arne Nordmann (noro),CC BY-SA 3.0

Newton's theory of light was preferred over Huygen's because **Newton had a very high reputation** at the time, also diffraction had not yet been observed and the speed of light hadn't been measured.

3.12.2.2 - Significance of Young's double slit experiment

In Young's double slit experiment **coherent light** is shone through 2 slits so that it diffracts; each slit acts as a **coherent point source** making a pattern of light and dark fringes. Light fringes are formed where the light meets **in phase** and **interferes constructively**, this occurs where the path difference between waves is a **whole number** of wavelengths ($n\lambda$, where n is an integer). Dark fringes are formed where the light meets **completely out of phase** and **interferes destructively**, this occurs where the path difference is a whole number and a half wavelengths ($(n+\frac{1}{2})\lambda$). \



If Newton's theory was correct, an interference pattern wouldn't be formed during the above experiment, instead there would only be **two bright fringes corresponding to the two slits** in the apparatus. Young's double slit experiment demonstrated **diffraction** and **interference** of light, which are both wave properties showing that Huygen's wave theory was in fact correct. Even after the Young's double slit experiment, which disproved corpuscular theory, **Huygen's theory wasn't widely accepted because Newton was a historical figure** who scientists expected to be correct. It wasn't until the **speed of light was measured in water** that Newton's theory was disregarded, because it was found that **light travels slower in water which contradicts the corpuscular theory of light**.

3.12.2.3 - Electromagnetic waves

Electromagnetic waves are formed of an alternating magnetic and electric fields travelling **in phase** and **at right angles to each other**. The direction of wave travel is **perpendicular** to the oscillations of the electric and magnetic fields.

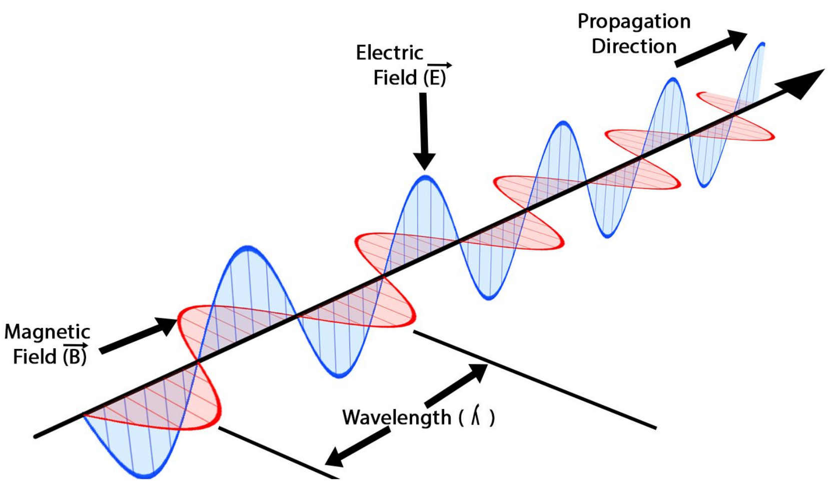


Image source: DECHAMMAKL,CC BY-SA 4.0

Maxwell predicted that EM waves existed and theorised a formula for their speed in a vacuum (c), before there was any experimental evidence for their existence.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Where μ_0 is the permeability of free space, and ε_0 is the permittivity of free space.
The permeability of free space (μ_0) relates the **magnetic flux density produced by a wire to the current in the wire**, and the permittivity of free space (ε_0) relates the **electric field strength to the charge on the object, which formed the field**.

Hertz discovered radio waves by using an apparatus which allowed **high voltage sparks to jump across a gap of air** as this leads to the production of radio waves. The radio waves could be detected by using a:

- **Dipole receiver** - this detects the waves' electric field. This is made by placing a second set of charged plates parallel to those forming the high voltage sparks.
- **A loop of wire with a gap** - this will detect the waves' alternating magnetic field as the field will enter the loop causing a change in magnetic flux, inducing a potential difference which will cause a spark to cross the gap in the wire.

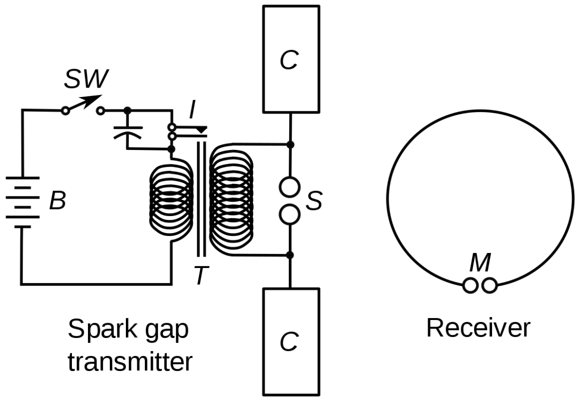


Image source: Original version: DMGualtieri, This version: Chetvorno ,CC BY-SA 3.0

By placing a **metal sheet** in front of the apparatus, the radio waves are reflected back onto themselves causing **stationary waves** to be formed. By using one of the detectors above, you can find the **distance between adjacent nodes** (points of no displacement) in order to find the wavelength, and using the frequency of the waves, calculate their speed. The speed calculated by Hertz was found to be the same as Maxwell's predicted value of the speed of electromagnetic waves, which helped confirm that radio waves were EM waves.

When the receiver is rotated about the line between the transmitter and detector, the **signal varies from a maximum value to a minimum after a rotation of 90°**, this is because at the maximum value, the plane of the detector is **perpendicular** to the oscillations of the electric/magnetic field however after a rotation of 90°, the plane of the detector is **parallel** to the oscillations of the field therefore no signal is detected. This showed that the produced radio waves were polarised.
Fizeau measured the speed of light using the following method:

1. A pulsed beam of light is passed through a gap in a toothed wheel rotating at a slow speed. The beam of light reflects on a mirror a large distance behind the wheel causing it to return back through the same gap between teeth in the wheel.
2. The **speed of rotation of the wheel is increased** until the **light beam can no longer be seen** because it is blocked by a tooth in the wheel next to the gap it could previously pass through. (If this **speed is doubled**, the **light would be visible** again because the light will now return through the gap next to the one it initially passes through).

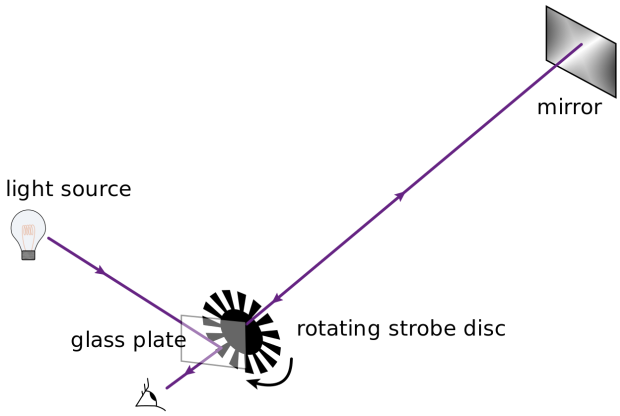


Image source: Fred the Oyster,CC BY-SA 4.0

Using the frequency that the wheel needs to travel at, and its number of teeth, it is possible to calculate the time taken by the light to travel to the mirror and back to the wheel:

For a wheel with 'n' teeth and 'n' gaps, after $\frac{1}{2n}$ of a revolution a tooth will replace a gap.

The time taken for one revolution is $\frac{1}{f}$ (where f is the frequency of the revolution), therefore a tooth will replace a gap every $\frac{1}{2nf}$ seconds.

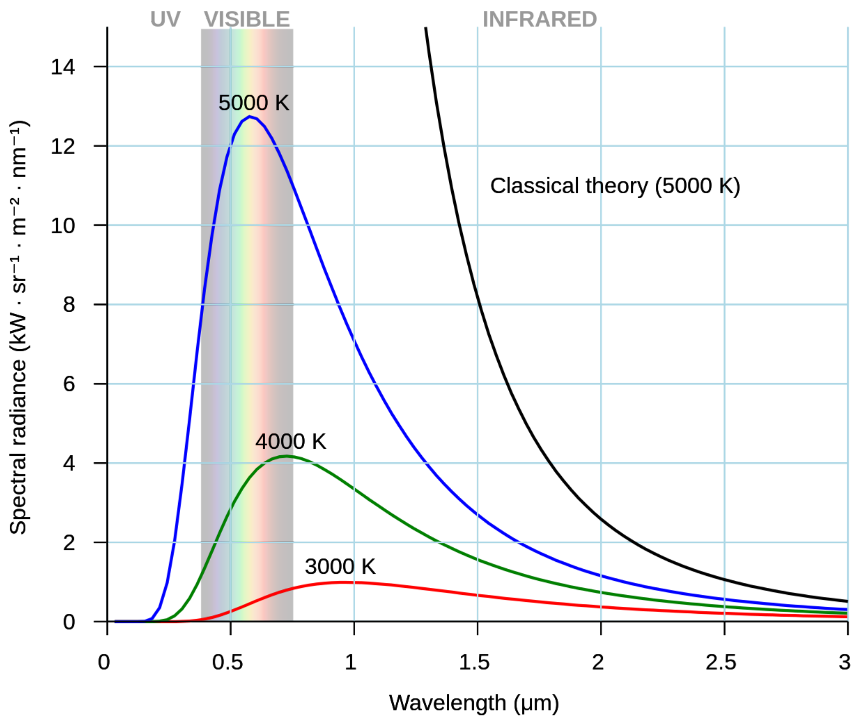
If the distance between the wheel and mirror is d, then the speed of light can be given by:

$$\frac{2d}{\frac{1}{2nf}} = 4dnf$$

Fizeau's result was extremely significant because it was very close to the value predicted by Maxwell, which **provided evidence that light was an EM wave**.

3.12.2.4 - The discovery of photoelectricity

A **black body** absorbs and emits all possible wavelengths of radiation. Wave theory predicted that as the wavelength of radiation decreases, the intensity of the radiation would increase, leading to a **prediction of infinite amount of ultraviolet radiation** being emitted, however this was not supported by experimental evidence. This lead to the **ultraviolet catastrophe**, because the widely accepted **wave theory predicted an impossible amount of UV radiation** and it c**ould not be used to explain experimental measurements**. Below is a graph of intensity against wavelength for the radiation emitted by a black body; the black line showing the classical prediction and the blue line showing the corresponding practical results.



The UV catastrophe could be resolved by using **Planck's interpretation of EM waves**: EM waves travel in **discrete packets** called **quanta**, which have an energy directly proportional to their frequency ($E = hf$).

The photoelectric effect also couldn't be explained by wave theory as:

1. **Wave theory suggests that any frequency of light should be able to cause photoelectric emission** as the energy absorbed by each electron will gradually increase with each incoming wave, and so can't explain the existence of a **threshold frequency**.
2. The **photoelectric effect is immediate**, which contradicts wave theory which suggests time is needed for the energy supplied to the electrons to reach the **work function** (minimum energy required for electrons to be emitted from the surface of a metal).
3. **Increasing the intensity** of the light does not increase the speed of photoelectric emission as would be suggested by wave theory, but instead it **increases the number of photoelectrons released per second**.
4. Photoelectrons are released with a **range of kinetic energies**.

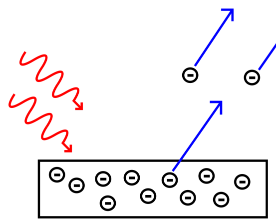


Image source: Wolfmankurd,CC BY-SA 3.0

Einstein expanded on Planck's work, by suggesting EM waves are released in discrete packets which he called photons, which have particle-like interactions. This model could be used to explain all the points above which wave theory couldn't:

1. When a photon interacts with an electron, **all of its energy is transferred to it**, and **an electron can only interact with a single photon**. If this energy is above the work function, a photoelectron is emitted, if this energy is below the work function, the electron remains in place. As the energy of a photon is directly proportional to frequency ($E = hf$), the **threshold frequency is the frequency at which the photon energy is equal to the work function of the metal**.
2. The photon energy is transferred to the electron immediately when they interact, leading to photoelectrons being emitted immediately.
3. **Intensity is equal to the number of photons released per second**, if this is increased the number of photoelectrons emitted is increased because **more photons interact with electrons per second**.
4. All electrons will receive the **same amount of energy** from a photon of light, however electrons which are deeper in the **metal will lose energy through collisions** when leaving the metal, and will therefore have a lower kinetic energy. Electrons will also need to do work if the surface of the metal is positively charged.

If a potential is applied across a metal surface that makes it **positive**, the **kinetic energy of the photoelectrons will decrease as they must do work against the electrostatic force of attraction towards the surface**. As this potential is increased, the number of photoelectrons released will decrease, because the number of photoelectrons with a high enough kinetic energy to be emitted decreases.
The **stopping potential** is the potential difference you would need to apply across the metal to **stop the photoelectrons with the maximum kinetic energy**. Measuring stopping potential allows you to find the maximum kinetic energy of the released photoelectrons, as $E_{k\,(max)} = eV_s$. Where V_s is the stopping potential and e is the charge of an electron. This is derived using the fact that $energy = charge \times voltage$.

You can substitute the above equation into Einstein's photoelectric equation:

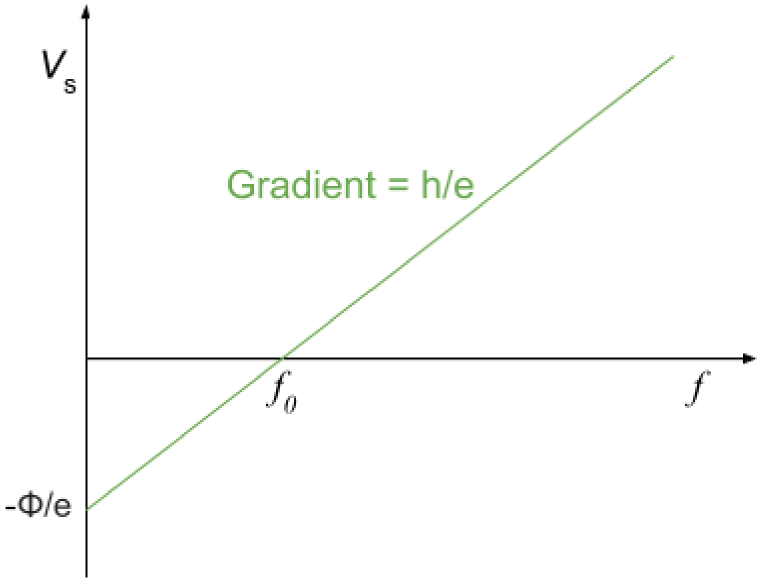
$$E = hf = \Phi + E_{k\,(max)}$$

$$hf = \phi + eV_s$$

Rearrange to make V_s the subject:

$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$

Therefore, if you plot a graph of **stopping potential against frequency** of light, you will get a straight line graph with a gradient of $\frac{h}{e}$, a y-intercept of $-\frac{\Phi}{e}$ and an x-intercept which is the **threshold frequency** of the metal, as shown on the diagram to the right. This experimental evidence confirmed Einstein's photon theory of light.



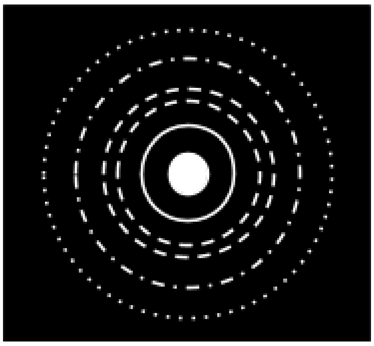
3.12.2.5 - Wave-particle duality

The **de Broglie hypothesis** states that **all particles have a wave-like nature and a particle nature**, and that the wavelength of any particle can be found using the following equation:

$$\lambda = \frac{h}{mv}$$ where h is the planck constant

As mv is also known as the momentum (p), the above equation can be re-written:

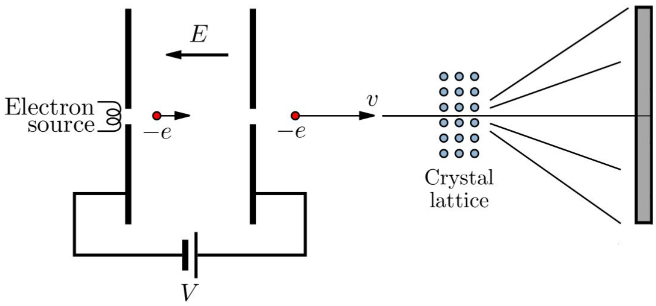
$$p = \frac{h}{\lambda}$$



The electron diffraction interference pattern forms concentric rings

Electron diffraction provided experimental evidence for the de Broglie hypothesis as it showed that electrons, which are particles, can also undergo diffraction, which can only be experienced by waves.

This was performed using an electron gun, which accelerated electrons through a vacuum tube towards a crystal lattice, where they **interacted with the small gaps between atoms** and formed a diffraction pattern on a fluorescent screen behind the crystal.



Using the fact that product of **accelerating voltage(V) and charge of an electron is equal to the kinetic energy of the electrons emitted by the particle accelerator**, you can derive a form of the above equation specific to this experiment:

$$\frac{1}{2}mv^2 = eV$$
$$m^2v^2 = 2meV$$

$$mv = \sqrt{2meV}$$
$$\lambda = \frac{h}{\sqrt{2meV}}$$

Using the above formula you can see that as the **accelerating voltage is increased**, the speed of electrons increases and so their **de Broglie wavelength decreases**, so the electrons are diffracted more and the **fringe spacing will decrease** (the rings will move closer). The converse is also true, as **accelerating voltage is decreased**, the speed of electrons decreases so their **de Broglie wavelength increases**, so the electrons are diffracted less and the **fringe spacing will increase.** This follows **wave theory**, which states the fringe spacing in a diffraction pattern will increase as wavelength increases, providing further evidence to support the de Broglie hypothesis.

3.12.2.6 - Electron microscopes

The **resolving power** of a microscope is its ability to distinguish structures which are close to each other. The **wavelength of an electron beam is much smaller than that of light**, meaning electron microscopes have a much higher **resolving power** than light microscopes, so are used to view incredibly small structures. As the **wavelength of the electrons decreases the resolving power of the microscope increases**.

There are two types of electron microscope:

- **Transmission electron microscope (TEM)**
  In a TEM electrons are accelerated by an electron gun, and pass through a set of magnetic lenses all of which have different purposes (outlined below), passing through an **extremely thin sample** so that the electrons do not slow down, and their wavelength doesn't change. All of the lenses will leave electrons at the centre of the lens undeflected, but will deflect electrons at the edge of the lens towards the axis (centre) of the microscope. A TEM consists of the following lenses:
  - ➔ **Condenser lens** - This is the first lens the electrons beams pass through and this lens **deflects the electrons so that they form a wide parallel beam**, which is directed at the sample.
  - ➔ **Objective lens** - This lens will **form an image of the sample**, which is directly above it.
  - ➔ **Projector lens** - This will **magnify the image made by the objective lens** and project it onto the fluorescent screen.

  If the **accelerating voltage of the electron gun is increased**, the speed of the electrons will increase, therefore their wavelength will decrease causing the **resolving power of the microscope to increase**, however the resolving power of a TEM is limited due to:
  - ➔ **Sample thickness** - as electrons pass through the sample they will slow down, causing their wavelength to increase and so the resolving power is decreased
  - ➔ **Electrons travelling at a range of speeds** - as the electron gun emits electrons through thermionic emission some electrons may lose kinetic energy while leaving the metal (due to collisions). This leads to electrons travelling at different speeds, having different wavelengths and therefore being diffracted by different amounts which causes blurring of the image (known as aberration).

In order to resolve details which are around the size of an atom, the de Broglie wavelength of the electron beam will have to be around the diameter of an atom (around 0.1 nm). You can calculate the accelerating voltage required for electrons to reach this wavelength by using the following formula (example shown below):

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$0.1 \times 10^{-9} = \frac{6.63\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times V}}$$ Rearrange for V

$$V = 150.8\text{ V}$$

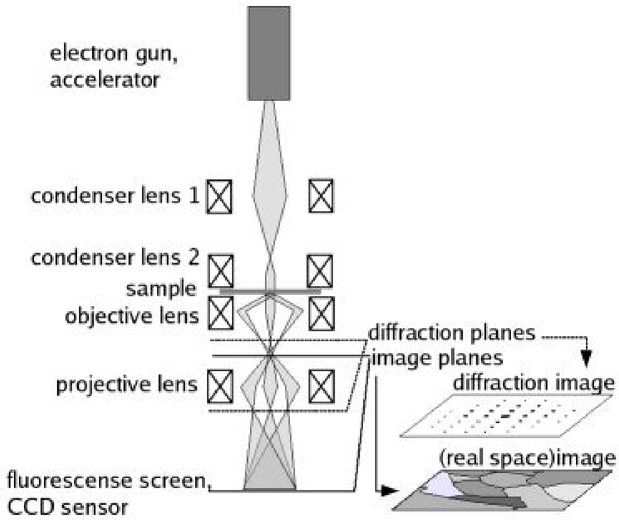
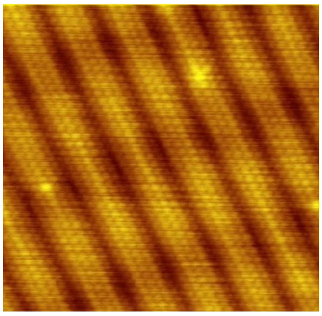


Image source: Ufalke,CC BY-SA 3.0

- **Scanning tunnelling microscope (STM)**
  An STM uses **quantum tunnelling of electrons** in order to form an image of the surface of an object. Quantum tunnelling occurs due to the **wave nature of electrons**, meaning that **if the barrier they are trying to cross (which could be physical or potential) is small enough electrons can move across it** just like light waves would be able to. The smaller the barrier/gap, the more likely it is that tunnelling will occur.



An STM is formed of a **very fine tipped probe**, which moves across the surface of an object and stays at a **constant potential** (can be either negative or positive), meaning electrons can only travel in one direction. This movement of electrons can be measured and is known as the **tunnelling current**. As the probe moves across the surface, the **size of the gap will vary**, when it becomes **larger**, **tunnelling current will decrease** as tunnelling of electrons is less likely to occur and when it becomes **smaller**, **tunnelling current will increase** as tunnelling of electrons is more likely.

There are two ways an STM can operate:

- ➔ **Constant height mode**
  The probe is kept at a constant height as it moves across the surface, and the tunnelling current is recorded and used to image the surface of the object.
- ➔ **Constant current mode**

The current is constantly monitored and fed back to the microscope allowing it to adjust the probe’s height so the current is kept constant. The movement of the probe can then be used to image the surface.

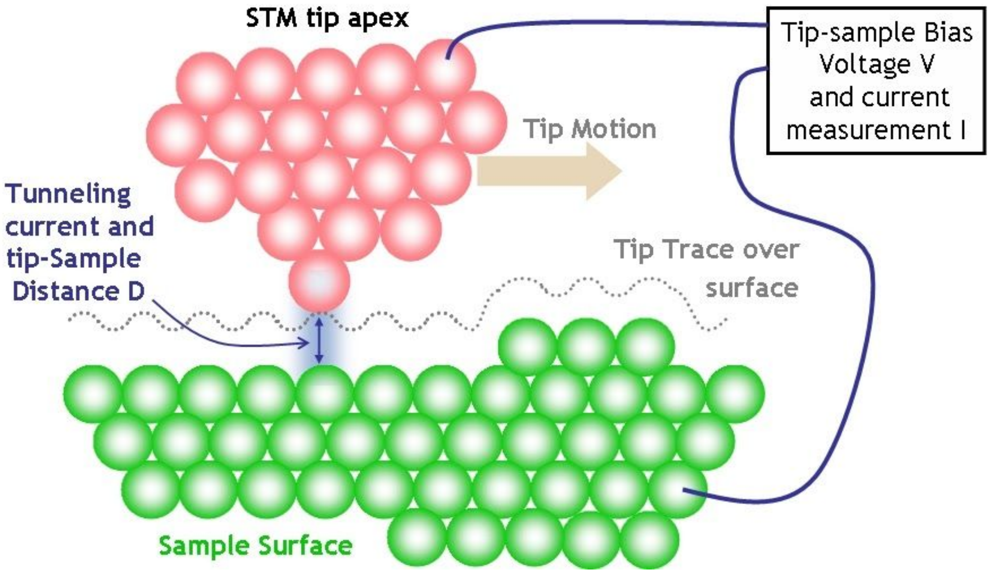


Image source: KristianMolhave,CC BY 2.5

3.12.3 Special relativity

3.12.3.1 - The Michelson-Morley experiment

Scientists used to believe in **absolute motion**, which is the idea that everything moves relative to the **ether,** which they believed was a substance which permeated the entire universe.

Michelson and Morley designed an apparatus known as an **interferometer** to measure the absolute speed of the Earth through the ether. The interferometer was formed of a partially reflective surface (beam splitter), a glass block (compensating plate) and two mirrors. The partially reflective surface would reflect some light while allowing some to pass through, creating two beams of light moving **perpendicular** to each other, which travel towards the two mirrors which are set up the **same distance** from the beam splitter. The glass block is used to make sure that the two beams of light **pass through the same amount of glass**. After being reflected on the mirrors, the two beams of light return to a detector and the interference pattern they form can be recorded.

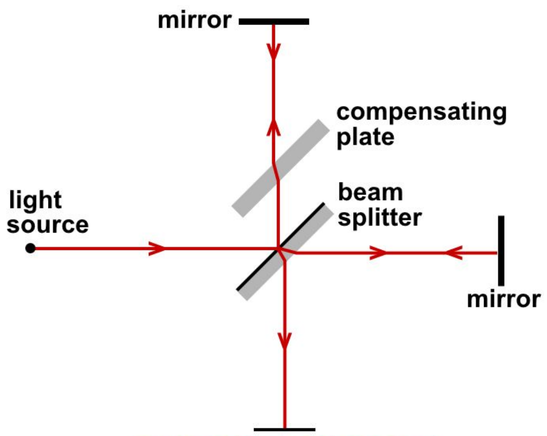


Image source: Stigmatella aurantiaca,CC BY-SA 3.0,
Image is cropped, and some text is removed

Michelson and Morely believed that the **speed of the light travelling parallel to the motion of the Earth would be affected**, while the speed of light travelling perpendicular to the motion of the Earth would be left unaffected. So it was also believed the **light moving parallel to the Earth's motion will take longer to travel**, therefore rotating the apparatus by 90° would cause a **shift in the interference pattern**. However, the interference pattern experienced no shift, showing that the **time taken for light to travel was unaffected by rotation of the apparatus**. There were 3 conclusions that could be drawn from this result:

1. The ether doesn't exist or the Earth drags the ether along with it as it moves
2. The speed of light is invariant in free space, meaning that the speed of light is independent of the motion of the source or the observer.

3.12.3.2 - Einstein's theory of special relativity

Inertial frames of reference are those which move at a constant velocity relative to each other, therefore a frame that is accelerating or rotating cannot be an inertial frame of reference.

Einstein's theory of **special relativity only applies to inertial frames** of reference and is based on the following postulates (assumptions):

1. The speed of light in free space is **invariant**
   - The speed of light is independent of the motion of the source or the observer
2. The **laws of physics have the same form in all inertial frames**
   - The laws of physics will act in the same way in all inertial frames of reference

3.12.3.3 - Time dilation

Time dilation is a consequence of special relativity, meaning it only occurs in inertial frames and causes time to run at different speeds depending on the motion of an observer.

The **stationary observer** is stationary relative to the frame of reference where an event is occurring, while to an **external observer** the frame of reference is in motion. The amount of time passed experienced by the stationary observer is known as the **proper time (t_0)**, whereas the time measured by an external observer can be denoted by **t**, and can be calculated using the following formula:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where v is the velocity of the at which the **stationary observer** is travelling.

It is important to remember that the **proper time will always be shorter than the time measured by an external observer**, as shown in the above equation as the denominator will always be less than or equal to zero.

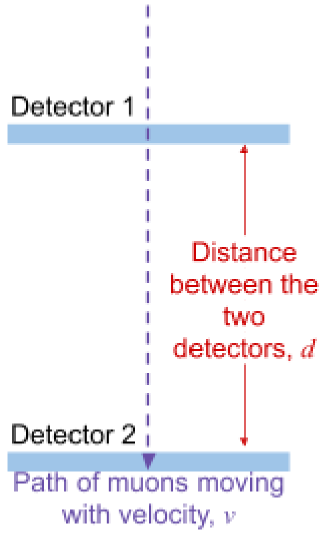
For example, Lucy is in a spaceship travelling at 0.9c and uses her clock to measure 1 hour, how much time would pass on Earth while she is measuring this time?

Lucy is the stationary observer in this example as she is stationary relative to the clock, while the Earth would be the external observer as it is in motion relative to the clock. So proper time (t_0) is 1 hour and the time passed on Earth is t, so the above equation can be used to calculate t:

$$t = \frac{1}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} \qquad t = \frac{1}{\sqrt{1 - 0.81}} \qquad t = 2.3 \text{ hours}$$

Muon decay provides **experimental evidence** for time dilation because muons enter the atmosphere at very high speeds and so experience significant time dilation, which affects how quickly they decay.

In order to measure muon decay you must place one detector at a high altitude and one much further down to measure the change in muon count rate, as shown in the diagram to the right. You will also have to measure the distance between the detectors (d) and the speed that they muons are travelling at (v).



Below are the results of such an experiment:

Count rate - at detector 1 - 100 s^{-1}
at detector 2 - 80 s^{-1}

Distance between the detectors, d - 2 km

Velocity of muons, v - 0.996c

Muon half-life (at rest) - 1.5 μs

Using the above data we can perform 2 sets of calculations:

1. **Expected count rate at 2nd detector** (**without acknowledging special relativity**)
   First, calculate the time taken for the muons to move between the detectors, using $t = \frac{s}{v}$:
   $$t = \frac{2\times10^3}{0.996\times3\times10^8} = 6.69\times10^{-6}\text{ s}$$
   Next, calculate the number of half-lives expected to occur in this time.
   $$\frac{6.69\times10^{-6}}{1.5\times10^{-6}} = 4.46$$
   The expected count rate will be the product of initial count and $(\frac{1}{2})^{4.46}$, as count rate decreases by half every half-life.
   $100\times(\frac{1}{2})^{4.46} =$ **4.5 s^{-1}** This is extremely **different from the observed count rate**, which shows that **the above calculation cannot be correct**.

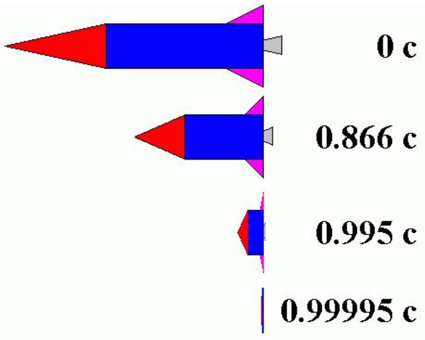
2. **Expected count rate at 2nd detector** (**acknowledging the effect special relativity**)
   First, calculate the time taken for the muons to move between the detectors from the external frame of reference (laboratory), using $t = \frac{s}{v}$:
   $$t = \frac{2\times10^3}{0.996\times3\times10^8} = 6.69\times10^{-6}\text{ s}$$
   Next, calculate the **proper time (t_0)**, which is actually experienced by the muons.
   $$t_0 = t\sqrt{1-\frac{v^2}{c^2}}$$
   $$t_0 = 6.69\times10^{-6}\times\sqrt{1-\frac{(0.996c)^2}{c^2}} = 6.0\times10^{-7}\text{ s}$$
   Next, calculate the number of half-lives expected to occur in this time.
   $$\frac{6.0\times10^{-7}}{1.5\times10^{-6}} = 0.4$$
   The expected count rate will be the product of initial count and $(\frac{1}{2})^{0.4}$, as count rate decreases by half every half-life.
   $100\times(\frac{1}{2})^{0.4} =$ **76 s^{-1}** As this value is **far closer to the observed value**, this experiment **provides evidence for the existence of special relativity**.

3.12.3.4 - Length contraction

Length contraction is a consequence of special relativity, meaning it only occurs in inertial frames and causes the length of objects moving at high speeds to appear shorter to an external observer.

The **proper length (l_0)** of an object is its length as **measured by an observer who is at rest relative to the object**. You can calculate the length of an object as seen by an external observer by using the following formula:

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$



Where v is the velocity of the at which the object is travelling.

It is important to note that even though the length of an object moving relative to an external observer will appear shorter, its **width will remain constant** as only length is affected.

Muon decay can also be used to provide experimental evidence for length contraction as well as time dilation. As the muons are travelling at such a high speed **the distance they travel will appear shorter than the distance as viewed by an external observer**. Using the data below, you can calculate the expected count rate considering the effect of length contraction:

Count rate - at detector 1 - 100 s^{-1}
at detector 2 - 80 s^{-1}

Distance between the detectors, d - 2 km

Velocity of muons, v - 0.996c

Muon half-life (at rest) - 1.5 μs

➔ **Expected count rate** (**considering length contraction**)

In this case, the proper length is 2 km because the scientists measuring this value are at rest relative to the distance between the two detectors, whereas the muons are in motion.

First, you must calculate the **length**, l, which is the length as it appears to the muons:

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$l = 2\times10^3\times\sqrt{1-\frac{(0.996c)^2}{c^2}} = 180\text{ m}$$

Next, find the time taken to travel this distance.

$$t = \frac{180}{0.996\times3\times10^8} = 6.0\times10^{-7}\text{ s}$$

Then, calculate the number of half-lives expected to occur in this time.

$$\frac{6.0\times10^{-7}}{1.5\times10^{-6}} = 0.4$$

The expected count rate will be the product of initial count and $(\frac{1}{2})^{0.4}$, as count rate decreases by half every half-life.

$100 \times (\frac{1}{2})^{0.4} =$ **76 s^{-1}**

As this value is **very close to the observed value**, this provides experimental evidence for length contraction.

Be very careful not to use both length contraction and time dilation when doing calculations like the one above, you can only consider one effect of special relativity at once because they are both interlinked.

3.12.3.5 - Mass and energy

In the theory of special relativity Einstein proved that **mass and energy are interchangeable** and can be related by the following equation:

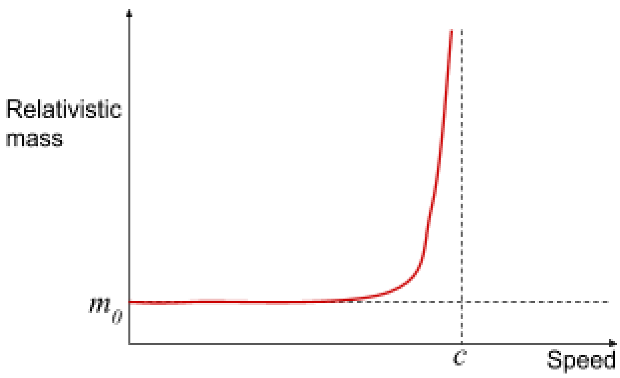
$$E = mc^2$$

Where E is energy, m is mass and c is the speed of light in a vacuum.

Transferring energy to an object will cause its mass to increase, while transferring energy away from an object will cause its mass to decrease. Because of this, the faster an object travels, the more massive it becomes. This larger value of mass is called the **relativistic mass (m)** and can be calculated using the following formula:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where v is the velocity of the at which the object is travelling, m_0 is rest mass and m is relativistic mass.



The classical calculation of kinetic energy by using the formula $\frac{1}{2}mv^2$ does not apply when objects are moving at **relativistic speeds (over 1/10th of the speed of light (c))**, because the mass of the object changes significantly. You can see in the graph of kinetic energy against speed below, that the classical calculation of kinetic energy, **KE$_{class}$** does not follow the observed values of kinetic energy, **KE$_{rel}$**.

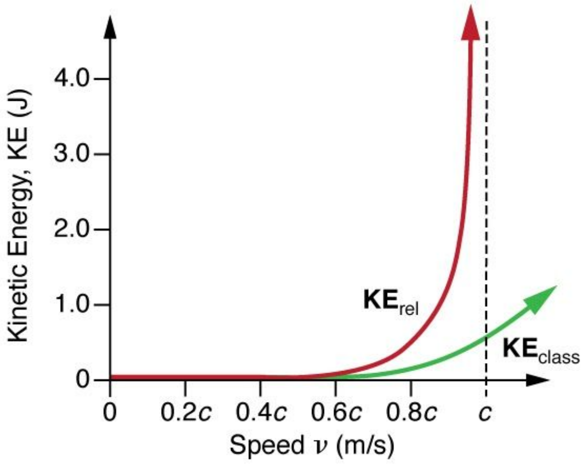


Image source: OpenStax College,CC BY 4.0

The total energy of a relativistic object can be calculated using the following formula:

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

As ***total energy (E$_T$) = kinetic energy (E$_k$) + rest energy (E$_0$)***, you can use the formula below to find the kinetic energy of an object moving at relativistic speeds:

$$E_k = E_T - E_0$$

$$E_k = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2$$

Where v is the velocity of the at which the object is travelling and m_0 is rest mass.

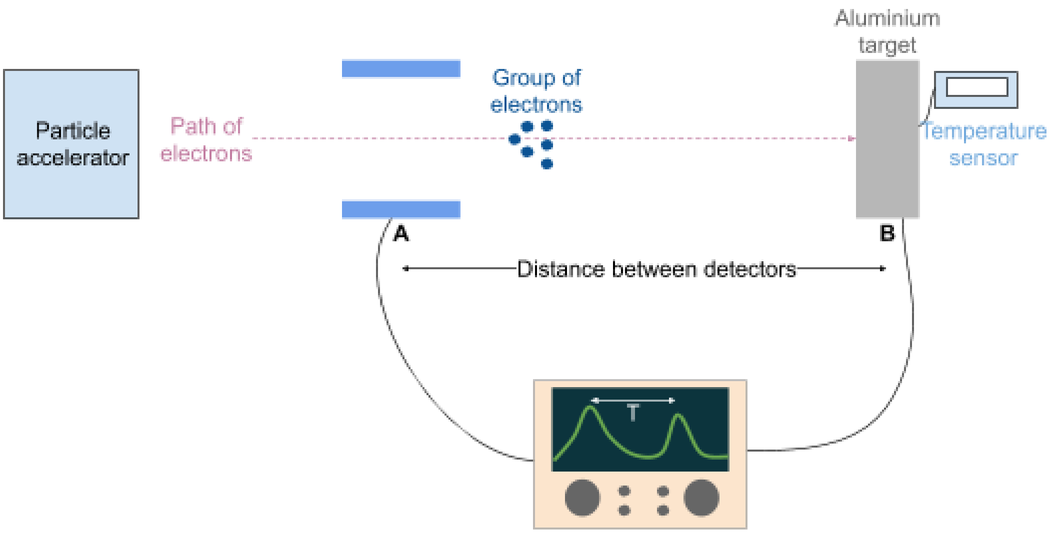
Bertozzi's experiment provides **experimental evidence for the increase in mass of an object with speed**. It involved a particle accelerator which could emit electrons at varying kinetic energies, two detectors, A and B, connected to an oscilloscope, and an aluminium plate connected to a temperature sensor:

1. The electrons were released in **pulses**, and the time taken for them to travel between A and B could be calculated using the oscilloscope by **measuring the distance between peaks** on the display (and multiplying by the time base).
2. Next, the distance between A and B is measured and the speed of the electrons was calculated.
3. The electrons are directed at the aluminium target and when they collide with it, **their kinetic energy is transferred to the target in the form of heat**. The change in

temperature of the target is measured using the temperature sensor, meaning the kinetic energy of the electrons could be directly measured.

Kinetic energy of one electron = $\frac{mc\Delta\theta}{n}$ Derived from: $Q = mc\Delta\theta$

Where m is the mass of the target, c is its specific heat capacity, Δθ is the change in temperature and n is the number of electrons in that pulse.



When Bertozzi plotted his results as a graph of kinetic energy against speed, he found that his **values were very close to those predicted by Einstein's theory of special relativity**, meaning that his experiment provided evidence to support it.

An important thing to note is that according to special relativity, **an object cannot reach the speed of light**. This is because as an object's speed approaches the speed of light, its mass approaches infinity and so **its energy also approaches infinity**, which is impossible as you cannot have an infinite amount of energy.

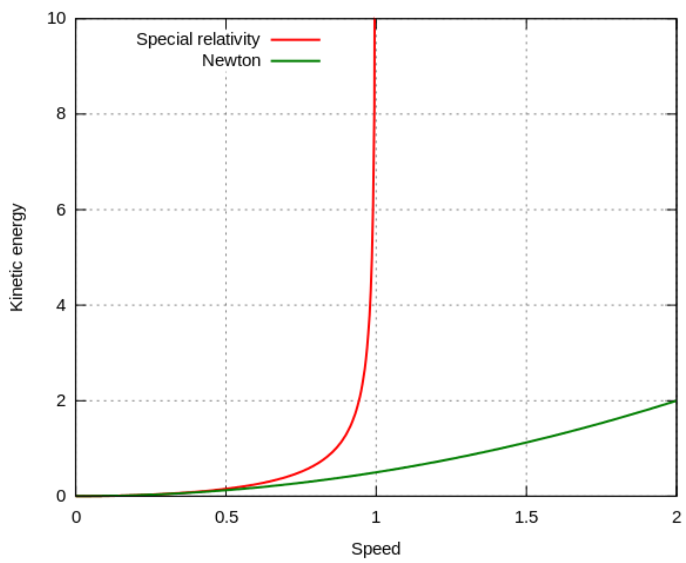


Image source: D.H,CC BY-SA 3.0