

Mathematics
Higher level
Paper 3 – discrete mathematics

Thursday 21 May 2015 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 14]

(a) The weights of the edges of a graph H are given in the following table.

	A	B	C	D	E	F	G
A	–	5	4	–	–	–	–
B	5	–	–	–	5	–	–
C	4	–	–	5	2	–	–
D	–	–	5	–	3	–	6
E	–	5	2	3	–	5	4
F	–	–	–	–	5	–	1
G	–	–	–	6	4	1	–

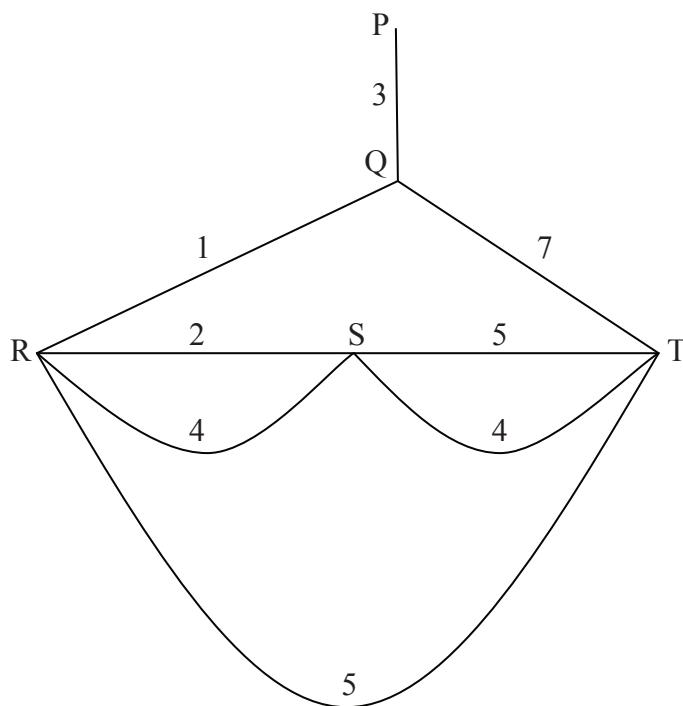
- (i) Draw the weighted graph H .
- (ii) Use Kruskal's algorithm to find the minimum spanning tree of H . Your solution should indicate the order in which the edges are added.
- (iii) State the weight of the minimum spanning tree.

[8]

(This question continues on the following page)

(Question 1 continued)

- (b) Consider the following weighted graph.



- (i) Write down a solution to the Chinese postman problem for this graph.
- (ii) Calculate the total weight of the solution. [3]
- (c) (i) State the travelling salesman problem.
- (ii) Explain why there is no solution to the travelling salesman problem for this graph. [3]

2. [Maximum mark: 7]

The graph $K_{2,2}$ is the complete bipartite graph whose vertex set is the disjoint union of two subsets each of order two.

- (a) Draw $K_{2,2}$ as a planar graph. [2]
- (b) Draw a spanning tree for $K_{2,2}$. [1]
- (c) Draw the graph of the complement of $K_{2,2}$. [1]
- (d) Show that the complement of any complete bipartite graph does not possess a spanning tree. [3]

3. [Maximum mark: 16]

The sequence $\{u_n\}$, $n \in \mathbb{N}$, satisfies the recurrence relation $u_{n+1} = 7u_n - 6$.

- (a) Given that $u_0 = 5$, find an expression for u_n in terms of n . [5]

The sequence $\{v_n\}$, $n \in \mathbb{N}$, satisfies the recurrence relation $v_{n+2} = 10v_{n+1} + 11v_n$.

- (b) Given that $v_0 = 4$ and $v_1 = 44$, find an expression for v_n in terms of n . [7]

- (c) Show that $v_n - u_n \equiv 15 \pmod{16}$, $n \in \mathbb{N}$. [4]

4. [Maximum mark: 12]

A simple connected planar graph, has e edges, v vertices and f faces.

- (a) (i) Show that $2e \geq 3f$ if $v > 2$. [6]

- (ii) Hence show that K_5 , the complete graph on five vertices, is not planar. [6]

- (b) (i) State the handshaking lemma. [4]

- (ii) Determine the value of f , if each vertex has degree 2. [4]

- (c) Draw an example of a simple connected planar graph on 6 vertices each of degree 3. [2]

5. [Maximum mark: 11]

- (a) State the Fundamental theorem of arithmetic for positive whole numbers greater than 1. [2]

- (b) Use the Fundamental theorem of arithmetic, applied to 5577 and 99 099, to calculate $\gcd(5577, 99\,099)$ and $\text{lcm}(5577, 99\,099)$, expressing each of your answers as a product of prime numbers. [3]

- (c) Prove that $\gcd(n, m) \times \text{lcm}(n, m) = n \times m$ for all $n, m \in \mathbb{Z}^+$. [6]