

**Mathematics**  
**Standard level**  
**Paper 2**

Thursday 12 November 2015 (afternoon)

Candidate session number

1 hour 30 minutes

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**Instructions to candidates**

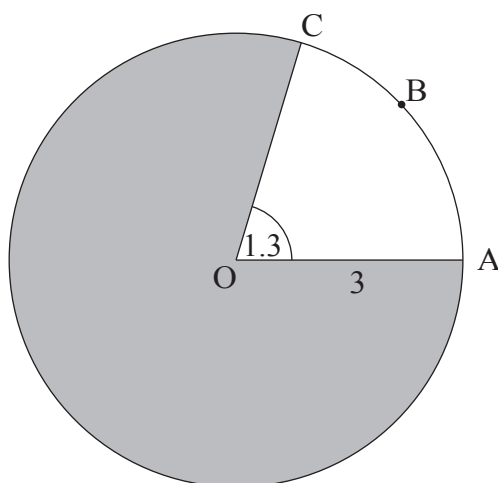
- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.



## Section A

**1.** [Maximum mark: 6]

**diagram not to scale**



(a) Find the length of arc ABC. [2]

(b) Find the area of the shaded region. [4]

[illegible]

2. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable  $X$ .

|            |      |     |     |      |
|------------|------|-----|-----|------|
| $x$        | 0    | 1   | 2   | 3    |
| $P(X = x)$ | 0.15 | $k$ | 0.1 | $2k$ |

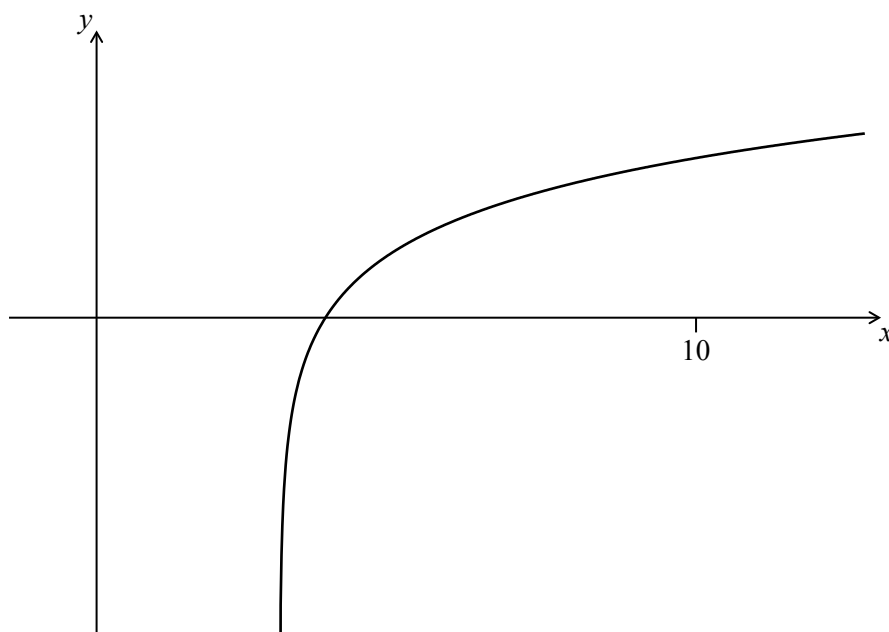
(a) Find the value of  $k$ . [3]

(b) Find  $E(X)$ . [2]



3. [Maximum mark: 7]

Let  $f(x) = 2 \ln(x - 3)$ , for  $x > 3$ . The following diagram shows part of the graph of  $f$ .



- (a) Find the equation of the vertical asymptote to the graph of  $f$ . [2]
- (b) Find the  $x$ -intercept of the graph of  $f$ . [2]
- (c) The region enclosed by the graph of  $f$ , the  $x$ -axis and the line  $x = 10$  is rotated  $360^\circ$  about the  $x$ -axis. Find the volume of the solid formed. [3]

(This question continues on the following page)



(Question 3 continued)

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Turn over

4. [Maximum mark: 7]

The first three terms of a geometric sequence are  $u_1 = 0.64$ ,  $u_2 = 1.6$ , and  $u_3 = 4$ .

(a) Find the value of  $r$ . [2]

(b) Find the value of  $S_6$ . [2]

(c) Find the least value of  $n$  such that  $S_n > 75\,000$ . [3]

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5. [Maximum mark: 7]

Let  $C$  and  $D$  be independent events, with  $P(C) = 2k$  and  $P(D) = 3k^2$ , where  $0 < k < 0.5$ .

(a) Write down an expression for  $P(C \cap D)$  in terms of  $k$ . [2]

(b) Given that  $P(C \cap D) = 0.162$ , find  $k$ . [2]

(c) Find  $P(C' | D)$ . [3]

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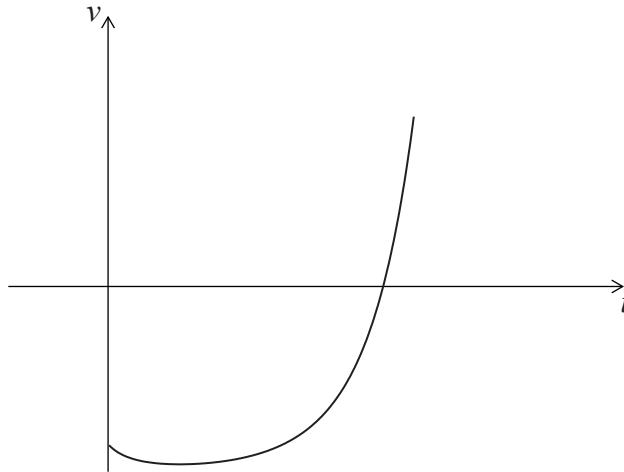


**6.** [Maximum mark: 6]

The velocity  $v \text{ m s}^{-1}$  of a particle after  $t$  seconds is given by

$$v(t) = (0.3t + 0.1)^t - 4, \text{ for } 0 \leq t \leq 5.$$

The following diagram shows the graph of  $v$ .



- (a) Find the value of  $t$  when the particle is at rest. [3]
- (b) Find the value of  $t$  when the acceleration of the particle is 0. [3]

[illegible]

7. [Maximum mark: 8]

Let  $f(x) = \ln(x^2)$ , for  $x \neq 0$ .

(a) Show that  $f'(x) = \frac{2}{x}$ . [2]

(b) The tangent to the graph of  $f$  at a point  $P(d, f(d))$  passes through another point  $Q(1, -3)$ . Find the value of  $d$ . [6]

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Do **not** write solutions on this page.

### Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The following diagram shows the quadrilateral ABCD.

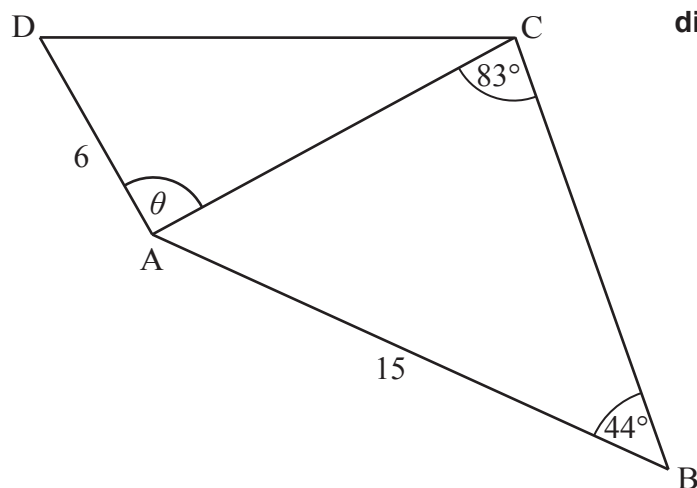


diagram not to scale

$AD = 6 \text{ cm}$ ,  $AB = 15 \text{ cm}$ ,  $\angle ABC = 44^\circ$ ,  $\angle ACB = 83^\circ$  and  $\angle DAC = \theta$

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

The area of triangle ACD is half the area of triangle ABC.

(c) Find the possible values of  $\theta$ . [5]

(d) Given that  $\theta$  is obtuse, find CD. [3]



Do **not** write solutions on this page.

9. [Maximum mark: 16]

An environmental group records the numbers of coyotes and foxes in a wildlife reserve after  $t$  years, starting on 1 January 1995.

Let  $c$  be the number of coyotes in the reserve after  $t$  years. The following table shows the number of coyotes after  $t$  years.

|                           |     |     |     |     |     |
|---------------------------|-----|-----|-----|-----|-----|
| number of years ( $t$ )   | 0   | 2   | 10  | 15  | 19  |
| number of coyotes ( $c$ ) | 115 | 197 | 265 | 320 | 406 |

The relationship between the variables can be modelled by the regression equation  $c = at + b$ .

- (a) Find the value of  $a$  and of  $b$ . [3]
- (b) Use the regression equation to estimate the number of coyotes in the reserve when  $t = 7$ . [3]

Let  $f$  be the number of foxes in the reserve after  $t$  years. The number of foxes can be modelled by the equation  $f = \frac{2000}{1 + 99e^{-kt}}$ , where  $k$  is a constant.

- (c) Find the number of foxes in the reserve on 1 January 1995. [3]
- (d) After five years, there were 64 foxes in the reserve. Find  $k$ . [3]
- (e) During which year were the number of coyotes the same as the number of foxes? [4]



Do **not** write solutions on this page.

10. [Maximum mark: 14]

The masses of watermelons grown on a farm are normally distributed with a mean of 10 kg. The watermelons are classified as small, medium or large.

A watermelon is small if its mass is less than 4 kg. Five percent of the watermelons are classified as small.

(a) Find the standard deviation of the masses of the watermelons.

[4]

The following table shows the percentages of small, medium and large watermelons grown on the farm.

| small | medium | large |
|-------|--------|-------|
| 5 %   | 57 %   | 38 %  |

A watermelon is large if its mass is greater than  $w$  kg.

(b) Find the value of  $w$ .

[2]

All the medium and large watermelons are delivered to a grocer.

(c) The grocer selects a watermelon at random from **this** delivery. Find the probability that it is medium.

[3]

(d) The grocer sells all the medium watermelons for \$1.75 each, and all the large watermelons for \$3.00 each. His costs on this delivery are \$300, and his total profit is \$150. Find the number of watermelons in the delivery.

[5]

