

**Mathematical studies**  
**Standard level**  
**Paper 1**

Tuesday 10 May 2016 (afternoon)

Candidate session number

1 hour 30 minutes

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**Instructions to candidates**

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.



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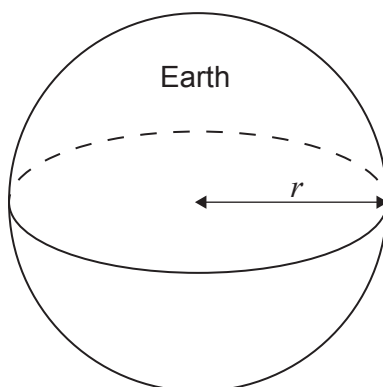
Answers written on this page will not  
be marked.



20EP02

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. Assume that the Earth is a sphere with a radius,  $r$ , of  $6.38 \times 10^3$  km.



- (a) (i) Calculate the surface area of the Earth in  $\text{km}^2$ .
- (ii) Write down your answer to part (a)(i) in the form  $a \times 10^k$ , where  $1 \leq a < 10$  and  $k \in \mathbb{Z}$ . [4]

The surface area of the Earth that is covered by water is approximately  $3.61 \times 10^8 \text{ km}^2$ .

- (b) Calculate the percentage of the surface area of the Earth that is covered by water. [2]

**Working:**

**Answers:**

- (a) (i) .....
- (ii) .....
- (b) .....



2. Consider the numbers  $-1$ ,  $4$ ,  $\frac{2}{3}$ ,  $\sqrt{2}$ ,  $0.35$  and  $-2^2$ .

Complete the following table by placing a tick (✓) to indicate if the number is an element of the number set. The first row has been completed as an example.

|               | N | Z | Q | R |
|---------------|---|---|---|---|
| $-1$          |   | ✓ | ✓ | ✓ |
| $4$           |   |   |   |   |
| $\frac{2}{3}$ |   |   |   |   |
| $\sqrt{2}$    |   |   |   |   |
| $0.35$        |   |   |   |   |
| $-2^2$        |   |   |   |   |

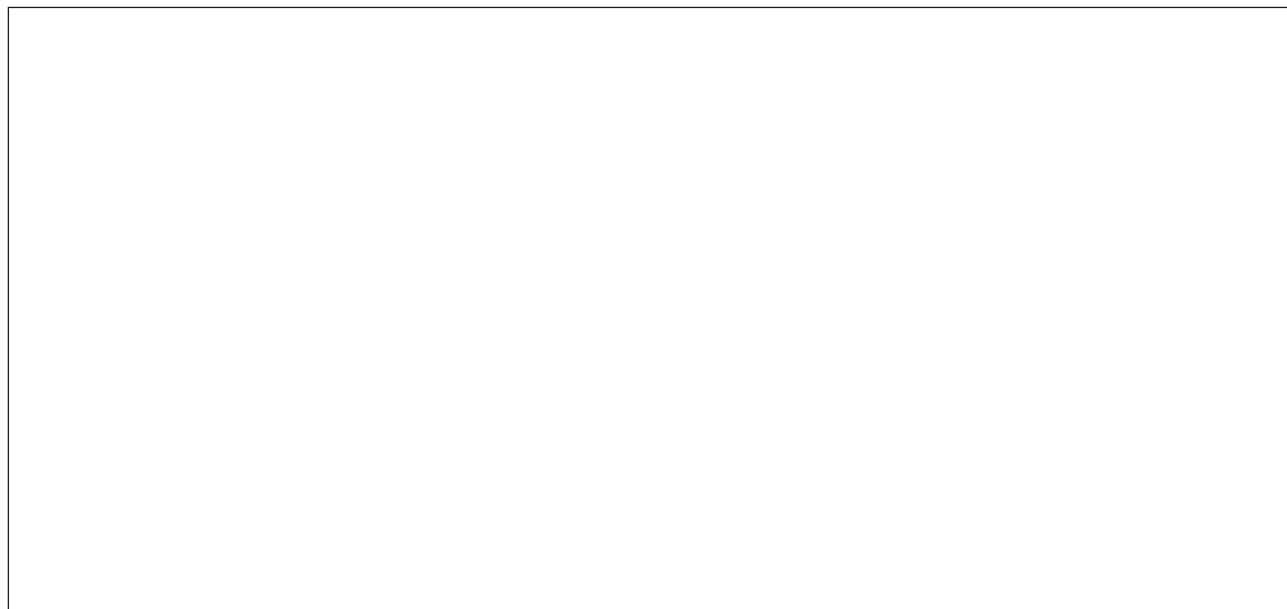
[6]



3. A ladder is standing on horizontal ground and leaning against a vertical wall. The length of the ladder is 4.5 metres. The distance between the bottom of the ladder and the base of the wall is 2.2 metres.

- (a) Use the above information to sketch a labelled diagram showing the ground, the ladder and the wall.

[1]



- (b) Calculate the distance between the top of the ladder and the base of the wall.

[2]

- (c) Calculate the **obtuse** angle made by the ladder with the ground.

[3]

**Working:**

**Answers:**

- (b) .....
- (c) .....



4. Consider the following propositions:

$p$ : The lesson is cancelled

$q$ : The teacher is absent

$r$ : The students are in the library.

(a) Write, in words, the compound proposition  $q \Rightarrow (p \wedge r)$ . [3]

(b) Complete the following truth table. [2]

| $q$ | $r$ | $\neg r$ | $q \Rightarrow \neg r$ |
|-----|-----|----------|------------------------|
| T   | T   |          |                        |
| T   | F   |          |                        |
| F   | T   |          |                        |
| F   | F   |          |                        |

(c) **Hence**, justify why  $q \Rightarrow \neg r$  is not a tautology. [1]

**Working:**

**Answers:**

(a) .....

(c) .....



5. Two friends, Sensen and Cruz, are conducting an investigation on probability.

Sensen has a fair six-sided die with faces numbered 1, 2, 2, 4, 4 and 4. Cruz has a fair disc with one red side and one blue side.

The die and the disc are thrown at the same time.

Find the probability that

- (a) the number shown on the die is 1 **and** the colour shown on the disc is blue; [2]
- (b) the number shown on the die is 1 **or** the colour shown on the disc is blue; [2]
- (c) the number shown on the die is even given that the colour shown on the disc is red. [2]

**Working:**

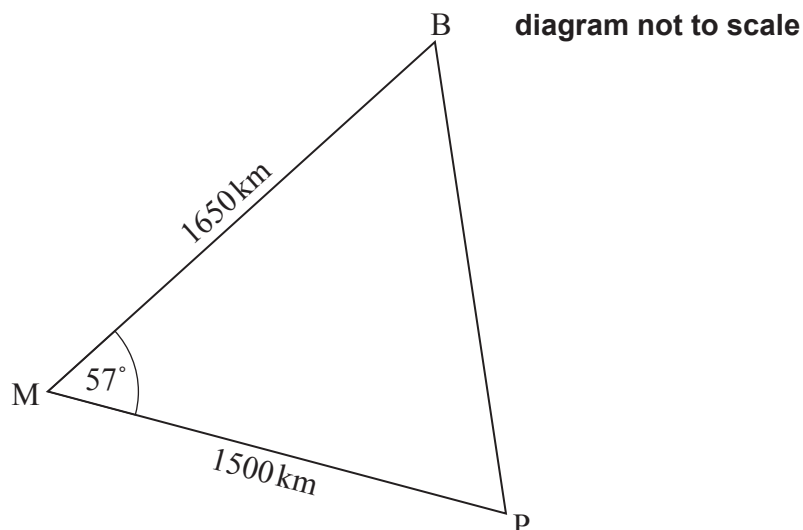
**Answers:**

- (a) .....
- (b) .....
- (c) .....



6. When Bermuda (B), Puerto Rico (P), and Miami (M) are joined on a map using straight lines, a triangle is formed. This triangle is known as the Bermuda triangle.

According to the map, the distance MB is 1650 km, the distance MP is 1500 km and angle BMP is  $57^\circ$ .



- (a) Calculate the distance from Bermuda to Puerto Rico, BP. [3]
- (b) Calculate the area of the Bermuda triangle. [3]

**Working:**

**Answers:**

- (a) .....
- (b) .....



7. A survey was conducted among a random sample of people about their favourite TV show. People were classified by gender and by TV show preference (Sports, Documentary, News and Reality TV).

The results are shown in the contingency table below.

|        | Sports | Documentary | News | Reality TV | Total |
|--------|--------|-------------|------|------------|-------|
| Male   | 20     | 24          | 32   | 11         | 87    |
| Female | 18     | 30          | 20   | 25         | 93    |
| Total  | 38     | 54          | 52   | 36         | 180   |

- (a) Find the expected number of females who prefer documentary shows. [2]

A  $\chi^2$  test at the 5% significance level is used to determine whether TV show preference is independent of gender.

- (b) Write down the  $p$ -value for the test. [2]

- (c) State the conclusion of the test. Give a reason for your answer. [2]

**Working:**

**Answers:**

- (a) .....
- (b) .....
- (c) .....



8. Consider the curve  $y = 1 + \frac{1}{2x}$ ,  $x \neq 0$ .

(a) For this curve, write down

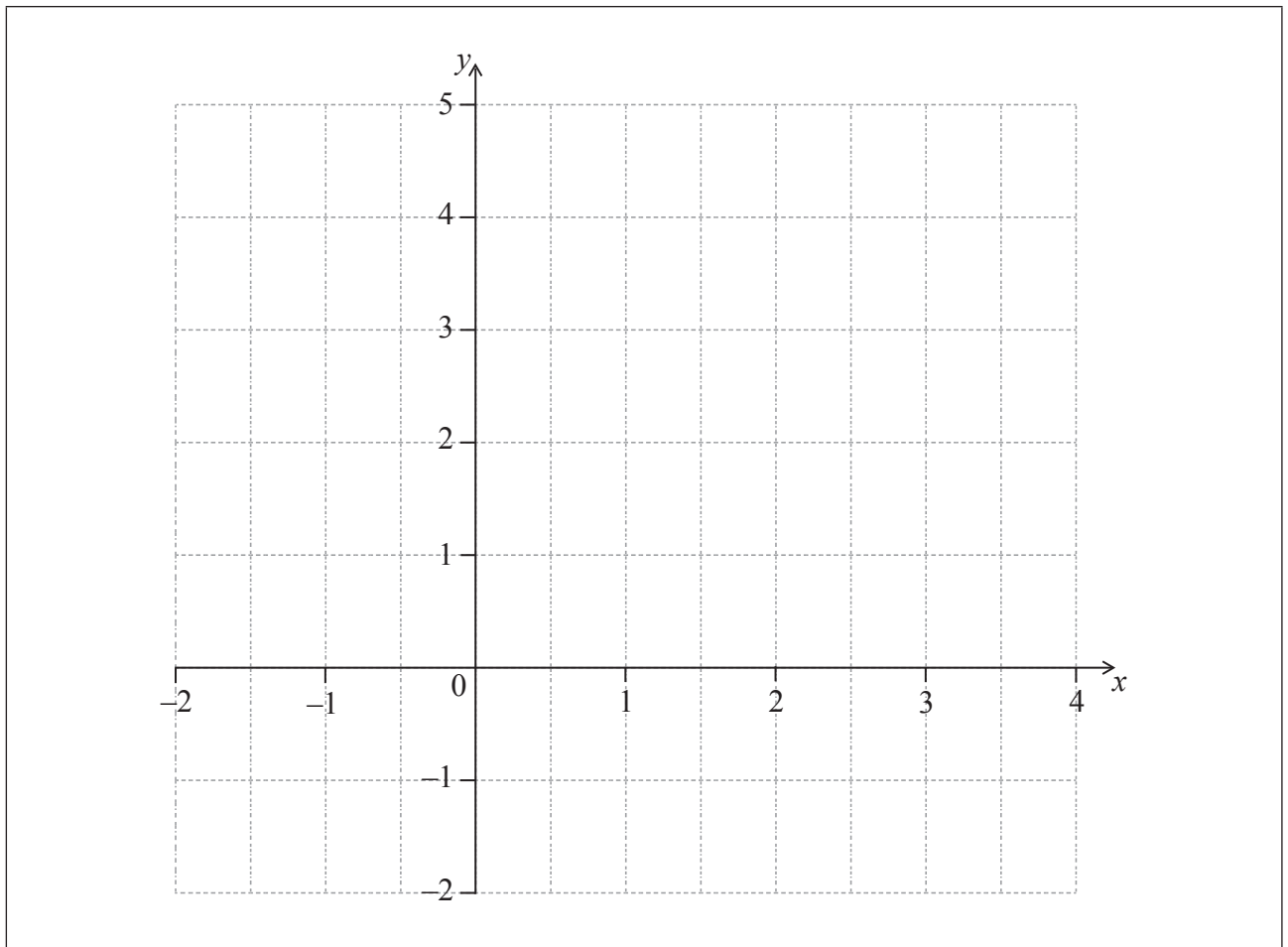
(i) the value of the  $x$ -intercept;

(ii) the equation of the vertical asymptote.

[3]

(b) Sketch the curve for  $-2 \leq x \leq 4$  on the axes below.

[3]



(This question continues on the following page)



(Question 8 continued)

**Working:**

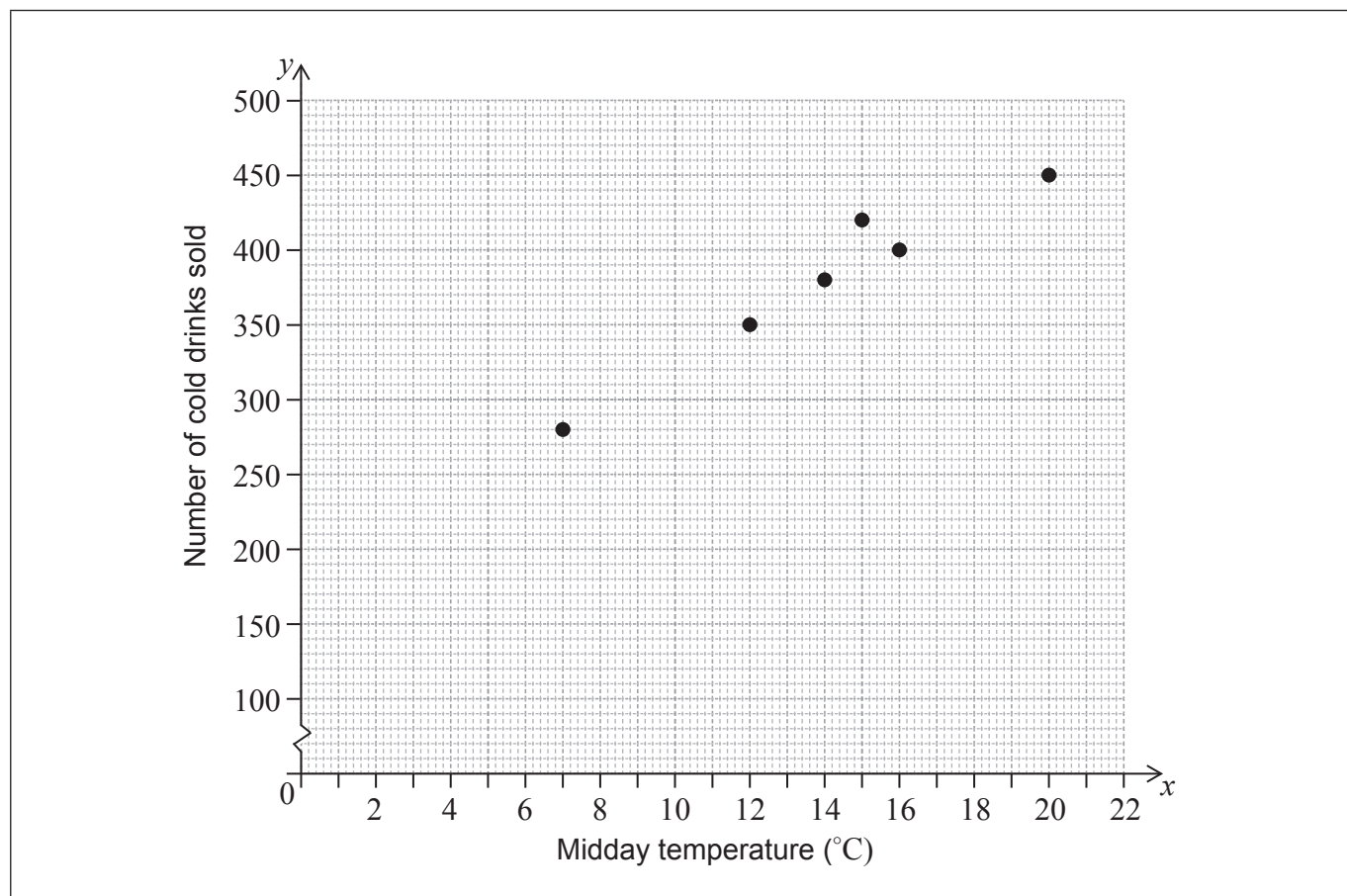
**Answers:**

- (a) (i) .....
- (ii) .....



9. Each day a supermarket records the midday temperature and how many cold drinks are sold on that day. The following table shows the supermarket's data for the last 6 days. This data is also shown on a scatter diagram.

|                                                    |     |     |     |     |     |     |
|----------------------------------------------------|-----|-----|-----|-----|-----|-----|
| <b>Midday temperature, °C (<math>x</math>)</b>     | 7   | 12  | 14  | 15  | 16  | 20  |
| <b>Number of cold drinks sold (<math>y</math>)</b> | 280 | 350 | 380 | 420 | 400 | 450 |



- (a) Write down
- (i) the mean temperature,  $\bar{x}$  ;
  - (ii) the mean number of cold drinks sold,  $\bar{y}$  .
- (b) Draw the line of best fit on the scatter diagram.
- (c) Use the line of best fit to estimate the number of cold drinks that are sold on a day when the midday temperature is 10°C.

[2]

[2]

[2]

(This question continues on the following page)



(Question 9 continued)

**Working:**

**Answers:**

- (a) (i) .....
- (ii) .....
- (c) .....



10. Obi travels from Dubai to Pretoria and changes 2000 United Arab Emirates Dirham (AED) at a bank. He receives 6160 South African Rand (ZAR).  
The exchange rate is 1 AED =  $x$  ZAR.

(a) Calculate the value of  $x$ .

[2]

Obi decides to invest half of the money he receives, 3080 ZAR, in an account which pays a nominal interest rate of 9%, **compounded monthly**.

The amount of money in the account will have doubled before the end of the  $n$ th year of the investment.

(b) Calculate the minimum value of  $n$ .

[4]

**Working:**

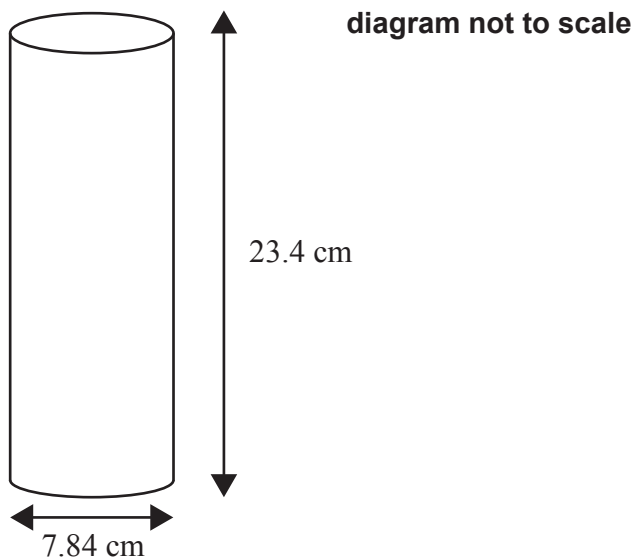
**Answers:**

(a) .....

(b) .....



11. A snack container has a cylindrical shape. The diameter of the base is 7.84 cm. The height of the container is 23.4 cm. This is shown in the following diagram.



(a) Write down the radius, in cm, of the base of the container. [1]

(b) Calculate the area of the base of the container. [2]

Dan is going to paint the curved surface and the base of the snack container.

(c) Calculate the area to be painted. [3]

**Working:**

**Answers:**

- (a) .....
- (b) .....
- (c) .....



12. The equation of the straight line  $L_1$  is  $y = 2x - 3$ .

(a) Write down the  $y$ -intercept of  $L_1$ . [1]

(b) Write down the gradient of  $L_1$ . [1]

The line  $L_2$  is parallel to  $L_1$  and passes through the point  $(0, 3)$ .

(c) Write down the equation of  $L_2$ . [1]

The line  $L_3$  is perpendicular to  $L_1$  and passes through the point  $(-2, 6)$ .

(d) Write down the gradient of  $L_3$ . [1]

(e) Find the equation of  $L_3$ . Give your answer in the form  $ax + by + d = 0$ , where  $a$ ,  $b$  and  $d$  are integers. [2]

**Working:**

**Answers:**

- (a) .....  
 (b) .....  
 (c) .....  
 (d) .....  
 (e) .....



13. A population of mosquitoes decreases exponentially. The size of the population,  $P$ , after  $t$  days is modelled by

$$P = 3200 \times 2^{-t} + 50, \text{ where } t \geq 0.$$

- (a) Write down the **exact** size of the initial population. [1]
- (b) Find the size of the population after 4 days. [2]
- (c) Calculate the time it will take for the size of the population to decrease to 60. [2]

The population will stabilize when it reaches a size of  $k$ .

- (d) Write down the value of  $k$ . [1]

**Working:**

**Answers:**

- (a) .....
- (b) .....
- (c) .....
- (d) .....



14. A group of students were asked how long they spend practising mathematics during the week. The results are shown in the following table.

| Time, $t$ (hours) | Number of students |
|-------------------|--------------------|
| $0 \leq t < 1$    | 35                 |
| $1 \leq t < 2$    | 30                 |
| $2 \leq t < 3$    | $a$                |
| $3 \leq t < 4$    | 52                 |
| $4 \leq t < 5$    | 43                 |

It is known that  $35 < a < 52$ .

(a) Write down

- (i) the modal class;
- (ii) the mid-interval value of the modal class;
- (iii) the class in which the median lies.

[3]

For this group of students, the estimated mean number of hours spent practising mathematics is 2.69.

(b) Calculate the value of  $a$ .

[3]

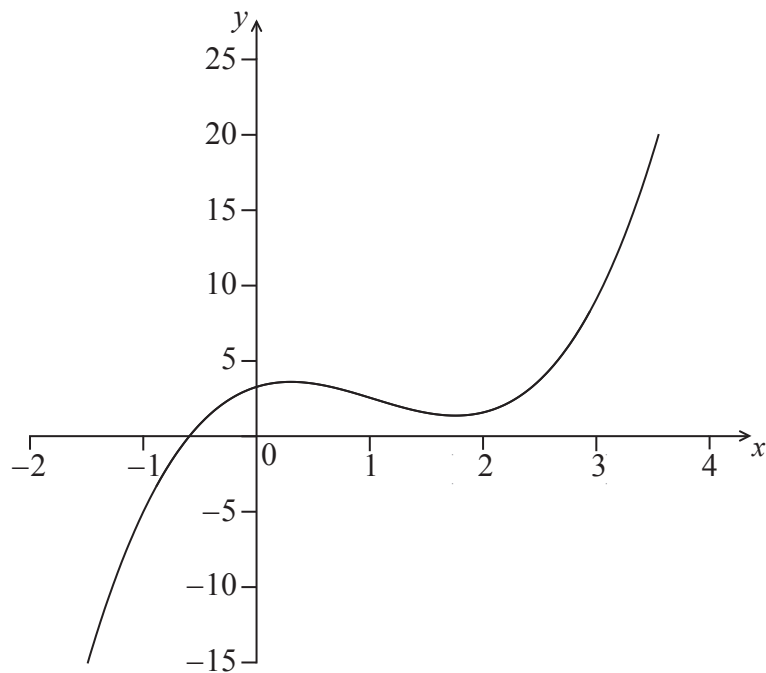
**Working:**

**Answers:**

- (a) (i) .....
- (ii) .....
- (iii) .....
- (b) .....



15. Consider the function  $f(x) = x^3 - 3x^2 + 2x + 2$ . Part of the graph of  $f$  is shown below.



- (a) Find  $f'(x)$ . [3]
- (b) There are two points at which the gradient of the graph of  $f$  is 11. Find the  $x$ -coordinates of these points. [3]

**Working:**

**Answers:**

(a) .....

(b) .....



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