

Mathematics
Higher level
Paper 3 – calculus

Wednesday 18 May 2016 (morning)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[60 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

The function f is defined by $f(x) = e^x \sin x$, $x \in \mathbb{R}$.

- (a) By finding a suitable number of derivatives of f , determine the Maclaurin series for $f(x)$ as far as the term in x^3 . [7]
- (b) Hence, or otherwise, determine the exact value of $\lim_{x \rightarrow 0} \frac{e^x \sin x - x - x^2}{x^3}$. [3]
- (c) The Maclaurin series is to be used to find an approximate value for $f(0.5)$.
 - (i) Use the Lagrange form of the error term to find an upper bound for the absolute value of the error in this approximation.
 - (ii) Deduce from the Lagrange error term whether the approximation will be greater than or less than the actual value of $f(0.5)$. [7]

2. [Maximum mark: 7]

A function f is given by $f(x) = \int_0^x \ln(2 + \sin t) dt$.

- (a) Write down $f'(x)$. [1]
- (b) By differentiating $f(x^2)$, obtain an expression for the derivative of $\int_0^{x^2} \ln(2 + \sin t) dt$ with respect to x . [3]
- (c) Hence obtain an expression for the derivative of $\int_x^{x^2} \ln(2 + \sin t) dt$ with respect to x . [3]

3. [Maximum mark: 9]

- (a) Given that $f(x) = \ln x$, use the mean value theorem to show that, for $0 < a < b$,

$$\frac{b-a}{b} < \ln \frac{b}{a} < \frac{b-a}{a}. \quad [7]$$

- (b) Hence show that $\ln(1.2)$ lies between $\frac{1}{m}$ and $\frac{1}{n}$, where m, n are consecutive positive integers to be determined. [2]

4. [Maximum mark: 13]

Consider the differential equation $\frac{dy}{dx} = \frac{x}{y} - xy$ where $y > 0$ and $y = 2$ when $x = 0$.

- (a) Show that putting $z = y^2$ transforms the differential equation into $\frac{dz}{dx} + 2xz = 2x$. [4]

- (b) By solving this differential equation in z , obtain an expression for y in terms of x . [9]

5. [Maximum mark: 14]

Consider the infinite series $S = \sum_{n=0}^{\infty} u_n$ where $u_n = \int_{n\pi}^{(n+1)\pi} \frac{\sin t}{t} dt$.

- (a) Explain why the series is alternating. [1]

- (b) (i) Use the substitution $T = t - \pi$ in the expression for u_{n+1} to show that $|u_{n+1}| < |u_n|$.

- (ii) Show that the series is convergent. [9]

- (c) Show that $S < 1.65$. [4]