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**Mathematics**  
**Higher level**  
**Paper 3 – statistics and probability**

Thursday 21 November 2019 (afternoon)

1 hour

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**Instructions to candidates**

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Peter, the Principal of a college, believes that there is an association between the score in a Mathematics test,  $X$ , and the time taken to run 500 m,  $Y$  seconds, of his students. The following paired data are collected.

|                             |     |     |    |     |    |     |
|-----------------------------|-----|-----|----|-----|----|-----|
| Mathematics test score $X$  | 70  | 75  | 76 | 66  | 60 | 61  |
| Time taken to run 500 m $Y$ | 100 | 105 | 95 | 109 | 89 | 101 |

It can be assumed that  $(X, Y)$  follow a bivariate normal distribution with product moment correlation coefficient  $\rho$ .

- (a) (i) State suitable hypotheses  $H_0$  and  $H_1$  to test Peter's claim, using a two-tailed test.
- (ii) Carry out a suitable test at the 5% significance level. With reference to the  $p$ -value, state your conclusion in the context of Peter's claim. [5]
- (b) Peter uses the regression line of  $y$  on  $x$  as  $y = 0.248x + 83.0$  and calculates that a student with a Mathematics test score of 73 will have a running time of 101 seconds. Comment on the validity of his calculation. [2]

2. [Maximum mark: 15]

- (a) Three independent random variables  $X_1, X_2, X_3$  are taken from a distribution with mean  $\mu$  and variance  $\sigma^2$ . Three estimators are proposed for  $\mu$ .

$$T_1 = \frac{X_1 + X_2 + X_3}{3}, T_2 = \frac{X_1 + 2X_2 + 3X_3}{3}, T_3 = \frac{X_1 + 2X_2}{3}$$

- (i) Show that one of these estimators for  $\mu$  is biased and show that the other two are unbiased.
- (ii) For the two unbiased estimators, determine, with a reason, which one is more efficient. [9]
- (b) Consider the random variable  $Y$ , which follows a negative binomial distribution  $Y \sim \text{NB}(4, p)$ . A random sample is taken from this distribution and the mean is denoted by  $\bar{Y}$ .
- (i) Find  $E(\bar{Y})$ .
- (ii) Hence suggest an unbiased estimator for  $\frac{1}{p}$  in terms of  $\bar{Y}$ . [2]

(This question continues on the following page)

**(Question 2 continued)**

- (c) A discrete random variable  $W$  has a probability distribution given by the following table.

|            |     |     |
|------------|-----|-----|
| $w$        | 1   | 2   |
| $P(W = w)$ | 0.5 | 0.5 |

- (i) Calculate  $E(W)$ .
- (ii) Calculate  $E\left(\frac{1}{W}\right)$ .
- (iii) Hence explain why your estimator for  $\frac{1}{p}$  in (b)(ii) does not directly suggest an unbiased estimator for  $p$ . [4]

**3.** [Maximum mark: 14]

- (a) State the central limit theorem as applied to a random sample of size  $n$ , taken from a distribution with mean  $\mu$  and variance  $\sigma^2$ . [2]

A random variable  $X$  has a distribution with mean  $\mu$  and variance 4. A random sample of size 100 is to be taken from the distribution of  $X$ .

- (b) Jack takes a random sample of size 100 and calculates that  $\bar{x} = 60.2$ . Find an approximate 90% confidence interval for  $\mu$ . [2]
- (c) Josie takes a different random sample of size 100 to test the null hypothesis that  $\mu = 60$  against the alternative hypothesis that  $\mu > 60$  at the 5% level.
- (i) Find the critical region for Josie's test, giving your answer correct to two decimal places.
- (ii) Write down the probability that Josie makes a Type I error.
- (iii) Given that the probability that Josie makes a Type II error is 0.25, find the value of  $\mu$ , giving your answer correct to three significant figures. [10]

4. [Maximum mark: 14]

Consider the random variable  $X$ , which follows a negative binomial distribution  $X \sim \text{NB}(r, p)$ . The probability generating function for  $X$  is given by

$$G_X(t) = \frac{p^r t^r}{(1 - qt)^r}, \text{ where } q = 1 - p.$$

- (a) Use this probability generating function to find and simplify  $E(X)$ . [5]

Consider another independent random variable  $Y$ , where  $Y \sim \text{NB}(s, p)$ .  
Let  $W = X + Y$ .

- (b) (i) Find the probability generating function for  $W$ .  
(ii) Hence identify the distribution that  $W$  follows and state its parameters.  
(iii) Given that  $r = 2$  and  $s = 3$ , calculate  $P(X = 3 | W = 7)$ . [9]
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