The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Cayley Paper

Thursday 16th March 2017

All candidates must be in *School Year 9 or below* (England and Wales), *S2 or below* (Scotland), or *School Year 10 or below* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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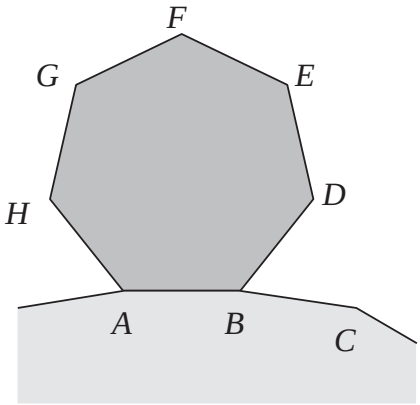
C1. Four times the average of two different positive numbers is equal to three times the greater one. The difference between the numbers is three less than the average.

What are the two numbers?

C2.



The diagram shows three adjacent vertices *A*, *B* and *C* of a regular polygon with forty-two sides, and a regular heptagon *ABDEFGH*. The polygons are placed together edge-to-edge.

Prove that triangle *BCD* is equilateral.

C3. Peaches spends exactly £3.92 on some fruit, choosing from apples costing 20p each and pears costing 28p each.

How many of each type of fruit might she have bought?

C4. The point *X* lies inside the square *ABCD* and the point *Y* lies outside the square, in such a way that triangles *XAB* and *YAD* are both equilateral.

Prove that $XY = AC$.

C5. In a sports league there are four teams and every team plays every other team once. A team scores 3 points for a win, 1 point for a draw, and 0 points for a loss.

What is the smallest number of points that a team could have at the end of the league and still score more points than each of the other teams?

C6. We write '*pq*' to denote the two-digit integer with tens digit *p* and units digit *q*.

For which values of *a*, *b* and *c* are the two fractions $\frac{\text{`}ab\text{'}}{\text{`}ba\text{'}}$ and $\frac{\text{`}bc\text{'}}{\text{`}cb\text{'}}$ equal and different from 1?