2017 Grey Kangaroo Solutions

1. C Since Florence is the fourth on the left from Jess, there are three girls between them going left round the circle. Similarly, since Florence is the seventh on the right from Jess, there are six girls between them going right. Therefore there are nine other girls in the circle apart from Florence and Jess. Hence there are 11 girls in total.

2. B When you evaluate correctly the left-hand side of each proposed equality in turn, you obtain 4, 2.5, 2, 1.75 and 1.6. Hence the only true equality is $\frac{5}{2} = 2.5$.

3. E The length of the outer rectangle is (3 + 4) m = 7 m longer than the length of the inner rectangle. The height of the outer rectangle is (2 + 3) m = 5 m longer than the height of the inner rectangle. Hence the length of the perimeter of the outer rectangle is $(2 \times 7 + 2 \times 5)$ m = 24 m longer than the length of the perimeter of the inner rectangle.

4. D The sum of the three smallest positive integers is $1 + 2 + 3 = 6$. Hence the only way to add three different positive integers to obtain a total of 7 is $1 + 2 + 4$. Therefore the product of the three integers is $1 \times 2 \times 4 = 8$.

5. B Since the areas of the four hearts are 1 cm^2, 4 cm^2, 9 cm^2 and 16 cm^2, the outer and inner shaded regions have areas $16\ cm^2 - 9\ cm^2 = 7\ cm^2$ and $4\ cm^2 - 1\ cm^2 = 3\ cm^2$ respectively. Therefore the total shaded area is $7\ cm^2 + 3\ cm^2 = 10\ cm^2$.

6. A Since $2017 = 33 \times 60 + 37$, a period of 2017 minutes is equivalent to 33 hours and 37 minutes or 1 day, 9 hours and 37 minutes. Hence the time 2017 minutes after 20:17 will be the time 9 hours and 37 minutes after 20:17, which is 05:54.

7. A The total amount of the money the five girls have is $(20 + 4 \times 10)$ euros = 60 euros. Therefore, if all five girls are to have the same amount, they need to have $(60 \div 5)$ euros = 12 euros each. Since each of Olivia's sisters currently has 10 euros, Olivia would need to give each of them $(12 - 10)$ euros = 2 euros.

8. E Adam the Ant has crawled $\frac{2}{3}$ of the length of the pole and so is $\frac{1}{3}$ of the length of the pole from the right-hand end. Benny the Beetle has crawled $\frac{3}{4}$ of the length of the pole and so is $\frac{1}{4}$ of the length of the pole from the left-hand end. Hence the fraction of the length of the pole that Adam and Benny are apart is $\left(1 - \frac{1}{3} - \frac{1}{4}\right) = \frac{5}{12}$.

9. C The ages of the four cousins are 3, 8, 12 and 14. When these are added in pairs, we obtain 3 + 8 = 11, 3 + 12 = 15, 3 + 14 = 17, 8 + 12 = 20, 8 + 14 = 22 and 12 + 14 = 26. Only two of these, 15 and 20, are divisible by 5. However, we are told that the sum of the ages of Alan and Dan and the sum of the ages of Carl and Dan are both divisible by 5. Hence, since Dan's age appears in both sums that are divisible by 5, his age is 12. Since Alan is younger than Carl, Alan's age is 3 and Carl's age is 8. Hence Bob's age is 14. Therefore the sum of the ages of Alan and Bob is 3 + 14 = 17.

10. A One sixth of the audience are adults. Therefore five sixths of the audience are children. Two fifths of the children are boys and hence three fifths of the children are girls. Therefore three fifths of five sixths of the audience are girls. Now $\frac{3}{5} \times \frac{5}{6} = \frac{1}{2}$. Hence the fraction of the audience who are girls is $\frac{1}{2}$.

11. E Since 35% of the entrants were female, 65% of the entrants were male. Hence, since there were 252 more males than females, 252 people represent $(65 - 35)\% = 30\%$ of the total number of entrants. Therefore the total number of entrants was $(252 \div 30) \times 100 = 840$.

12. D Since the sum of the numbers in the first three boxes is to be 22, the sum of the numbers in the last three boxes is to be 25 and the sum of the numbers in all five boxes is to be 35, Ellie will write $(22 + 25 - 35) = 12$ in the middle box. Therefore she will write $(22 - 3 - 12) = 7$ in the second box and $(25 - 12 - 4) = 9$ in the fourth box. Hence the product of the numbers in the shaded boxes is $7 \times 9 = 63$.

13. B Rohan wants to obtain 9 equal pieces and so makes eight marks. Jai wants to obtain 8 equal pieces and so makes seven marks. Since 9 and 8 have no common factors (9 and 8 are co-prime), none of the marks made by either boy coincide. Therefore Yuvraj will cut at 15 marked points and hence will obtain 16 pieces of string.

14. B Let the height of the lower triangle be h cm. Therefore the height of the upper triangle is $(8 - h)$ cm. Hence the shaded area in cm^2 is $\frac{1}{2} \times 1 \times h + \frac{1}{2} \times 1 \times (8 - h) = \frac{1}{2} \times (h + 8 - h) = 4$.

15. C Whichever day of the week Margot chooses for her first jogging day, there are four other days she can choose for her second day since she does not want to jog on either the day before or the day after her first chosen day. Therefore there are $7 \times 4 = 28$ ordered choices of days. However, the order of days does not matter when forming the timetable, only the two days chosen. Hence Margot can prepare $28 \div 2 = 14$ different timetables.

16. D Label the numbers Ella writes down as shown in the diagram.

2	a	b
c	d	3
e	f	g

Since the sum of the numbers in any two adjacent cells is the same, $2 + a = a + b$ and hence $b = 2$. Therefore $b + 3 = 2 + 3 = 5$. Hence the sum of the numbers in any two adjacent cells is 5. It is now straightforward to see that $a = c = f = 3$ and that $b = d = e = g = 2$. Therefore the sum of all the numbers in the grid is $5 \times 2 + 4 \times 3 = 22$.

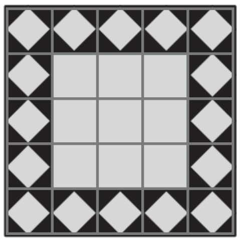
17. B Since 5 and 2 differ by 3, Tom can obtain the same integer from two different integers in the first list that also differ by 3 by adding 5 to the smaller integer and adding 2 to the larger integer. In the first list there are six pairs of integers that differ by 3, namely 1 and 4, 2 and 5, 3 and 6, 4 and 7, 5 and 8 and 6 and 9. However, the integers 4, 5 and 6 appear in two of these pairs and hence the same integer in the second list can be obtained from only three pairs of integers from the first list leaving three integers in the first list unpaired. Therefore, the smallest number of different integers Tom can obtain in the second list is six.

18. C Label the kangaroos facing right as K1, K2, K3, K4, K5 and K6 as shown in the diagram.

No further exchanges will be possible only when the kangaroos facing right have moved past all the kangaroos facing left. Kangaroos K1, K2 and K3 each have four left-facing kangaroos to move past while kangaroos K4, K5 and K6 each have two left-facing kangaroos to move past. Hence there will be $(3 \times 4 + 3 \times 2) = 18$ exchanges made before no further exchanges are possible.

19. A Since the car takes 35 minutes to travel from the airport to the city centre and the buses all take 60 minutes, the car will arrive 25 minutes before the bus it left with. Since buses leave the airport every 3 minutes, they will also arrive at the city centre every 3 minutes. Since $25 = 8 \times 3 + 1$, in the 25 minute spell between the car arriving and the bus it left with arriving eight other buses will arrive. Therefore the car overtook eight airport buses on its way to the city centre.

20. D Divide the tablecloth into 25 equal squares as shown. Half of each of the 16 outer squares is coloured black which is equivalent to 8 complete squares. Therefore the percentage of the tablecloth that is coloured black is $\frac{8}{25} \times 100 = 32$.

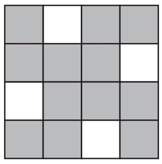


21. E Continue the sequence as described to obtain 2, 3, 6, 8, 8, 4, 2, 8, 6, 8, 8, 4, 2, 8, 6, 8 and so on. Since the value of each term depends only on the preceding two terms, it can be seen that, after the first two terms, the sequence 6, 8, 8, 4, 2, 8 repeats for ever. Now $2017 - 2 = 335 \times 6 + 5$. Therefore the 2017th number in the sequence is the fifth number of the repeating sequence 6, 8, 8, 4, 2, 8. Hence the required number is 2.

22. D Each of the nine tunnels in Stan's cube is five cubes long. However, the three tunnels starting nearest to the top front vertex of the cube all intersect one cube in. Similarly, the three tunnels starting at the centres of the faces all intersect at the centre of the large cube and the final three tunnels all intersect one cube in from the other end to that shown. Hence the number of small cubes not used is $9 \times 5 - 3 \times 2 = 45 - 6 = 39$.

23. E The ratio of the times taken to complete a circuit by Eric and Eleanor is 4:5. Therefore, since distance = speed × time and they both complete the same circuit, the ratio of their speeds is 5:4. Hence, since the total distance Eric and Eleanor cover between consecutive meetings is a complete circuit, Eleanor will run $\frac{4}{4+5}$ of a circuit between each meeting. Therefore Eleanor will run $\frac{4}{9}$ of 720 m which is 320 m between each meeting.

24. A Consider the four cells in the top left corner. It is not possible for all four cells to be coloured or the top left cell would not be touching an uncoloured cell and so there is at least one uncoloured cell in that group of four cells. By a similar argument, there is at least one uncoloured cell amongst the four cells in the bottom left corner, amongst the four cells in the bottom right corner and amongst the four cells in the top right corner. Therefore there are at least four uncoloured cells in the grid and hence at most twelve coloured cells. The diagram above shows that an acceptable arrangement is possible with twelve coloured cells.



Hence the largest number of cells Ellen can colour is twelve.

25. D The area of parallelogram $WXYZ$ is S. Therefore the area of triangle WXM, which has the same base and height, is $\frac{1}{2}S$. Hence the sum of the areas of triangle WMZ and triangle XYM is also $\frac{1}{2}S$. The sum of the areas of triangle WNZ and triangle XYP is given as $\frac{1}{3}S$ and therefore the sum of the areas of triangle ZNM and triangle MPY is $\frac{1}{2}S - \frac{1}{3}S = \frac{1}{6}S$. The area of triangle ZOY, which has the same base as the parallelogram but only half the height is $\frac{1}{2} \times \frac{1}{2}S = \frac{1}{4}S$. Therefore the area of quadrilateral $MNOP$ is $\frac{1}{4}S - \frac{1}{6}S = \frac{1}{12}S$.