United Kingdom Mathematics Trust

Grey Kangaroo

Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	C	E	D	B	A	D	A	E	C	B	E	B	B	C	D	C	B	D	C	B	A	A	E	B

1. Beate rearranges the five numbered pieces shown to display the smallest possible nine-digit number. Which piece does she place at the right-hand end?

A [4] B [8] C [31] D [59] E [107]

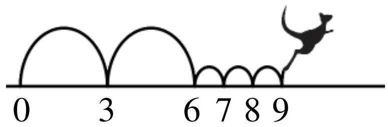
Solution **B**

The smallest nine-digit number is obtained by Beate choosing a piece for the left-hand end whose first digit is as small as possible and then repeating this process. Therefore the pieces are arranged in the order

[107][31][4][59][8]

and hence she places the piece with 8 on it at the right-hand end.

2. Kanga likes jumping on the number line. She always makes two large jumps of length 3, followed by three small jumps of length 1, as shown, and then repeats this over and over again. She starts jumping at 0. Which of these numbers will Kanga land on?



A 82 B 83 C 84 D 85 E 86

Solution **C**

Each time she completes a set of five jumps, Kanga moves forward 9 places on the number line. Since she started at 0, this means she will eventually land on $9 \times 9 = 81$. Her next set of jumps will take her to 84, 87, 88, 89 and 90. Therefore, of the numbers given, the only one Kanga will land on is 84.

3. The front number plate of Max's car fell off. He put it back upside down but luckily this didn't make any difference. Which of the following could be Max's number plate?

A [04 NSN 40] B [80 BNB 08] C [03 HNH 30] D [08 XBX 80]
E [60 HOH 09]

Solution **E**

Neither a "4" nor a "3" will look the same when turned upside down. The same is true about the letter "B". However, the letters "H" and "O" and the number "0" do look the same. The number "6" looks like a "9" when turned upside down and vice versa. Therefore the only number plate shown which would look the same if fitted upside down is [60 HOH 09].

4. In the equation on the right there are five empty squares. Sanja wants to fill four of them with plus signs and one with a minus sign so that the equation is correct.

$$6 \square 9 \square 12 \square 15 \square 18 \square 21 = 45$$

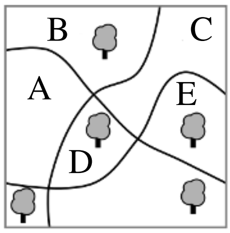
Where should she place the minus sign?

A Between 6 and 9 B Between 9 and 12 C Between 12 and 15
D Between 15 and 18 E Between 18 and 21

Solution **D**

The value of $6+9+12+15+18+21$ is 81. Now $81-45 = 36 = 2 \times 18$. Therefore, Sanja needs to subtract rather than add 18 and hence the minus sign should be placed between 15 and 18.

5. There are five big trees and three paths in a park. It has been decided to plant a sixth tree so that there are the same number of trees on either side of each path.
In which region of the park should the sixth tree be planted?



A B C D E

Solution **B**

The path running from the top of the park to the bottom has two trees to the left of it and three trees to the right of it on the diagram. Hence the sixth tree should be planted to the left of this path. The path running from the top left of the park to the bottom right has two trees above it and three trees below it on the diagram. Hence the sixth tree should be planted above this path. When we combine these observations, we can see that the sixth tree should be planted in the region labelled B. Note: this would also mean that there were the same number of trees on either side of the third path.

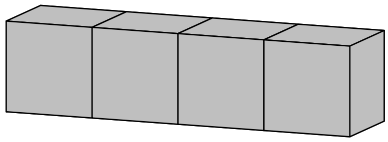
6. How many positive integers between 100 and 300 have only odd digits?

A 25 B 50 C 75 D 100 E 150

Solution **A**

For each digit to be odd, the first digit has to be 1, the second digit can be any one of 1, 3, 5, 7 or 9 and so can the third digit. Hence the number of positive integers between 100 and 300 with only odd digits is $1 \times 5 \times 5 = 25$.

7. On a standard dice, the sum of the numbers of pips on opposite faces is always 7. Four standard dice are glued together as shown. What is the minimum number of pips that could lie on the whole surface?



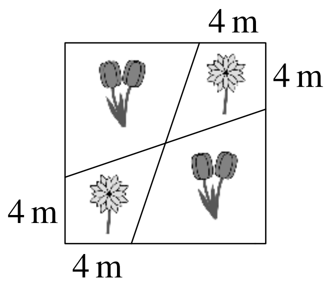
A 52 B 54 C 56 D 58 E 60

Solution **D**

Since the sum of the numbers of the pips on opposite faces is 7, the sum of the numbers of pips on the top and bottom faces of each dice is 7 as is the sum of the numbers of pips on the front and the back faces of each dice. To obtain the minimum number of pips on the surface, the dice should be arranged so that there is a 1 showing on both the left- and right-hand ends of the shape. Therefore the minimum number of pips that could lie on the whole surface is $4 \times 7 + 4 \times 7 + 1 + 1 = 58$.

8. Tony the gardener planted tulips  and daisies  in a square flowerbed of side-length 12 m, arranged as shown.

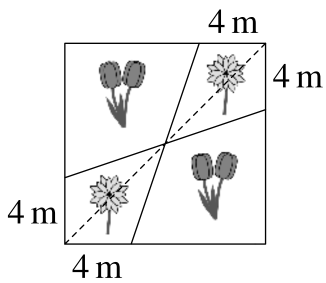
What is the total area, in m^2, of the regions in which he planted daisies?



A 48 B 46 C 44 D 40 E 36

Solution **A**

First consider the intersection point of the lines forming the boundaries of the regions containing daisies. Since the arrangement of the regions is symmetric, these lines intersect at the mid-point of the flowerbed. Therefore, the diagonal passes through the intersection point. It divides the daisy beds into four congruent triangles, each of base 4 m and height 6 m as shown. Hence the total area of the regions in which daisies are grown is, in m^2, equal to $4 \times \frac{1}{2} \times 4 \times 6 = 48$.



9. Three sisters, whose average age is 10, all have different ages. The average age of one pair of the sisters is 11, while the average age of a different pair is 12. What is the age of the eldest sister?

A 10 B 11 C 12 D 14 E 16

SOLUTION **E**

Since the average age of the three sisters is 10, their total age is $3 \times 10 = 30$.
Since the average age of one pair of the sisters is 11, their total age is $2 \times 11 = 22$ and hence the age of the sister not included in that pairing is $30 - 22 = 8$. Similarly, since the average age of a different pair of sisters is 12, their total age is $2 \times 12 = 24$ and hence the age of the sister not included in that pairing is $30 - 24 = 6$. Therefore the age of the eldest sister is $30 - 8 - 6 = 16$.

10. In my office there are two digital 24-hour clocks. One clock gains one minute every hour and the other loses two minutes every hour. Yesterday I set both of them to the same time but when I looked at them today, I saw that the time shown on one was 11:00 and the time on the other was 12:00.
What time was it when I set the two clocks?

A 23:00 B 19:40 C 15:40 D 14:00 E 11:20

SOLUTION **C**

Since one clock gains one minute each hour and the other clock loses two minutes each hour, for each hour that passes the difference between the times shown by the two clocks increases by three minutes. Therefore the amount of time in hours that has passed since the clocks were set is $60 \div 3 = 20$. In 20 hours, the clock that gains time will have gained 20 minutes. Hence the time at which the clocks were set is 20 hours and 20 minutes before 12:00 and so is 15:40.

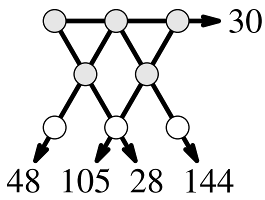
11. Werner wrote a list of numbers with sum 22 on a piece of paper. Ria then subtracted each of Werner's numbers from 7 and wrote down her answers. The sum of Ria's numbers was 34.
How many numbers did Werner write down?

A 7 B 8 C 9 D 10 E 11

SOLUTION **B**

For each number n that Werner wrote down, Ria wrote $7 - n$. Therefore, the sum of one of Werner's numbers and Ria's corresponding number is 7. Since the total of all Werner's numbers and all of Ria's numbers is $22 + 34 = 56$, the number of numbers that Werner wrote down is $56 \div 7 = 8$.

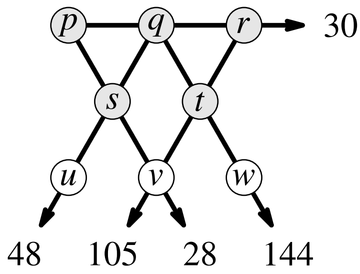
12. The numbers 1 to 8 are to be placed, one per circle, in the circles shown. The number next to each arrow shows what the product of the numbers in the circles on that straight line should be.
What will be the sum of the numbers in the three circles at the bottom of the diagram?



A 11 B 12 C 15 D 16 E 17

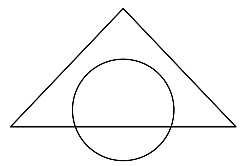
Solution **E**

Let the numbers in each of the circles be p, q, r, s, t, u, v and w, as shown in the diagram. Since the only two lines of numbers with products divisible by 5 meet at the circle containing letter r, we have $r = 5$. Similarly, since the only two lines of numbers with products divisible by 7 meet at the circle containing letter v, we have $v = 7$. Now consider the line of numbers with product 28. Since we know $v = 7$, we have $p \times s = 4$ and, since the numbers are all different, p and s are some combination of 1 and 4.



Now note that 4 is not a factor of 30 and so p cannot be 4 and hence $p = 1$ and $s = 4$. It is now easy to see that the only way to complete the diagram is to put $q = 6$, $t = 3$, $u = 2$ and $w = 8$. Therefore the sum of the numbers in the bottom three circles is $2 + 7 + 8 = 17$.

13. The area of the intersection of a triangle and a circle is 45% of the total area of the diagram. The area of the triangle outside the circle is 40% of the total area of the diagram. What percentage of the circle lies outside the triangle?



A 20% B 25% C 30% D $33\frac{1}{3}$% E 35%

Solution **B**

The area of the circle inside the triangle is 45% of the total area of the diagram. The area of the circle outside the triangle is $(100 - 40 - 45)\% = 15\%$ of the total area of the diagram. Therefore, the percentage of the circle that lies outside the triangle is $\frac{15}{15+45} \times 100 = 25\%$.

14. Jenny decided to enter numbers into the cells of a 3×3 table so that the sum of the numbers in all four possible 2×2 cells will be the same. The numbers in three of the corner cells have already been written, as shown.
Which number should she write in the fourth corner cell?

2		4
?		3

A 0 B 1 C 4 D 5 E 6

Solution **B**

Let the numbers in the centre left cell and the centre right cell be x and y and let the number in the lower left corner be z, as shown in the diagram. Since the sum of the numbers in all four possible 2×2 cells should be the same, by considering the top left 2×2 cell and the top right 2×2 cell, since the top two cells in the central column are common, we have $2 + x = 4 + y$ and hence $x = y + 2$.

2		4
x		y
z		3

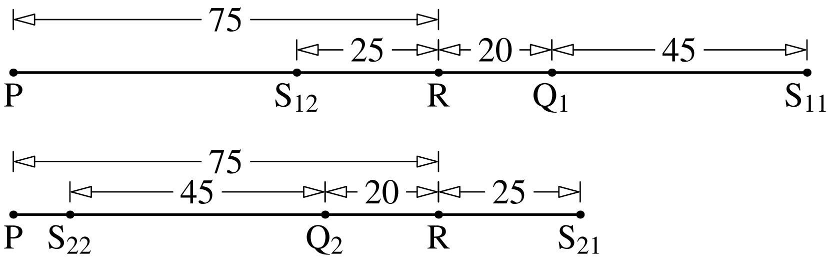
Similarly by considering the bottom left 2×2 cell and the bottom right 2×2 cell where the lower two cells in the central column are common, we have $z + x = y + 3$ and hence $z + y + 2 = y + 3$, which has solution $z = 1$. Therefore the value of the number in the fourth corner cell is 1.

15. The villages P, Q, R and S are situated, not necessarily in that order, on a long straight road. The distance from P to R is 75 km, the distance from Q to S is 45 km and the distance from Q to R is 20 km. Which of the following could **not** be the distance, in km, from P to S?

A 10 B 50 C 80 D 100 E 140

Solution **C**

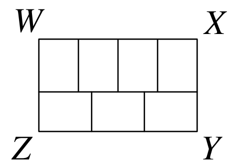
Since the distance from P to R is 75 km and the distance of Q from R is 20 km, there are two possible distances of Q from P, $(75 + 20)$ km = 95 km and $(75 - 20)$ km = 55 km. For each of the possible positions of Q, there are two possible positions of S, each 45 km from Q, as shown in the diagrams below.



Therefore the possible distances, in km, of S from P are $95 + 45 = 140$, $95 - 45 = 50$, $55 + 45 = 100$ and $55 - 45 = 10$. Therefore, of the options given, the one which is not a possible distance in km of S from P is 80.

16. The large rectangle $WXYZ$ is divided into seven identical rectangles, as shown.
What is the ratio $WX : XY$?

A $3:2$ B $4:3$ C $8:5$ D $12:7$ E $7:3$



Solution **D**

Let the longer side of each of the small rectangles be p and let the shorter side be q. From the diagram, it can be seen that $3p = 4q$ and hence $q = \frac{3}{4}p$. It can also be seen that the ratio $WX : XY = 3p : p + q$. This is equal to $3p : p + \frac{3}{4}p = 3p : \frac{7}{4}p = 12p : 7p = 12 : 7$.

17. You can choose four positive integers X, Y, Z and W. What is the maximum number of odd numbers you can obtain from the six sums $X+Y, X+Z, X+W, Y+Z, Y+W$ and $Z+W$?

A 2 B 3 C 4 D 5 E 6

Solution **C**

The sum of any two even integers is even and the sum of any two odd integers is also even. To obtain an odd number when adding two integers, one must be odd and one must be even. In a set of four integers, if one is odd and three are even there would be three possible sums of two integers that gave an odd number. Similarly, if one is even and three are odd there would also be three possible sums of two integers that gave an odd number. Also, if two of the four integers are odd and two are even, there would be $2 \times 2 = 4$ possible pairings that gave an odd answer. However, if all four integers are odd or if all four integers are even, there would be no possible sums of two integers that gave an odd answer. Hence the maximum number of odd numbers that could be obtained is 4.

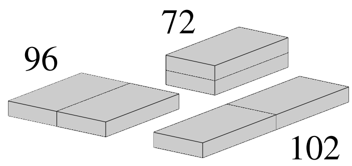
18. Marc always cycles at the same speed and he always walks at the same speed. He can cover the round trip from his home to school and back again in 20 minutes when he cycles and in 60 minutes when he walks. Yesterday Marc started cycling to school but stopped and left his bike at Eva's house on the way before finishing his journey on foot. On the way back, he walked to Eva's house, collected his bike and then cycled the rest of the way home. His total travel time was 52 minutes.
What fraction of his journey did Marc make by bike?

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{1}{2}$

Solution **B**

Let the fraction of his journey that Marc cycles be k. Therefore, the time he spends cycling is $20k$ and the time he spends walking is $60(1-k)$. Since the total time he takes is 52 minutes, we have $52 = 20k + 60(1-k)$ and hence $52 = 20k + 60 - 60k$. This simplifies to $8 = 40k$ which has solution $k = \frac{1}{5}$.

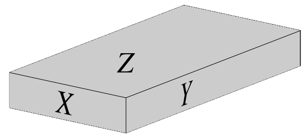
19. A builder has two identical bricks. She places them side by side in three different ways, as shown. The surface areas of the three shapes obtained are 72, 96 and 102. What is the surface area of the original brick?



A 36 B 48 C 52 D 54 E 60

Solution **D**

Let the areas of the front, the side and the top of the bricks be X, Y and Z, as shown in the diagram. From the question, we see that $4X + 4Y + 2Z = 72$, $4X + 2Y + 4Z = 96$ and $2X + 4Y + 4Z = 102$. When you add these three equations together you obtain $10X + 10Y + 10Z = 270$ and hence the surface area of the brick is $2X + 2Y + 2Z = 270 \div 5 = 54$.



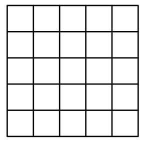
20. Carl wrote a list of 10 distinct positive integers on a board. Each integer in the list, apart from the first, is a multiple of the previous integer. The last of the 10 integers is between 600 and 1000. What is this last integer?

A 640 B 729 C 768 D 840 E 990

Solution **C**

The sequence of integers will have the form $q, qr, qrs, \ldots, qrstuvwxyz$ with the first integer q being successively multiplied by integers $r, s, \ldots, z$. The 10th integer in Carl's sequence, F, is then given by the product $F = qrstuvwxyz$. Since the 10 integers are all distinct, none of $r, s, t, \ldots, z$ is 1. Since $r, s, t, \ldots, z$ are integers, they are all at least 2 and hence $F \geq 2^9 = 512$. Therefore $q = 1$ or we would have $F \geq 2^{10} = 1024 > 1000$. All of $r, s, t, \ldots, z$ cannot be 2 since this would give $F = 2^9 = 512 < 600$. However, only one of $r, s, t, \ldots, z$ can be greater than 2 as otherwise we would have $F \geq 2^7 \times 3^2 = 1152 > 1000$. Hence $q = 1$, eight of the nine integers $r, s, t, \ldots, z$ are 2 and only one of them is greater than 2. That integer must be 3 since otherwise $F \geq 1 \times 2^8 \times 4 = 1024 > 1000$. Therefore the last integer in Carl's sequence is $1 \times 2^8 \times 3 = 256 \times 3 = 768$.

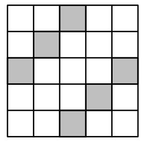
21. What is the smallest number of cells that need to be coloured in a 5×5 square grid so that every 1×4 or 4×1 rectangle in the grid has at least one coloured cell?



A 5 B 6 C 7 D 8 E 9

Solution **B**

For every 1×4 or 4×1 rectangle in the grid to have at least one coloured cell, there must be at least one coloured cell in every row and in every column. However, only one coloured cell in each row and column would not be sufficient as, for example, a coloured cell in the far right column and no other coloured cell in the same row as that cell would leave a 4×1 rectangle consisting of the other four cells in that row without a coloured cell in it.



Hence, any row or column in which an end cell is coloured must have at least one more coloured cell in it. Therefore at least six cells must be coloured and the diagram shows that such an arrangement is possible.
Note —many other arrangements of coloured cells also exist.

22. Mowgli asked a snake and a tiger what day it was. The snake always lies on Monday, Tuesday and Wednesday but tells the truth otherwise. The tiger always lies on Thursday, Friday and Saturday but tells the truth otherwise. The snake said “Yesterday was one of my lying days”. The tiger also said “Yesterday was one of my lying days”. What day of the week was it?

A Thursday B Friday C Saturday D Sunday E Monday

Solution **A**

The snake would only say “Yesterday was one of my lying days” on Monday, when it would be a lie and on Thursday, when it would be the truth. Similarly, the tiger would only say “Yesterday was one of my lying days” on Thursday, when it would be a lie, and on Sunday, when it would be the truth. Hence, since both said this, it was Thursday.

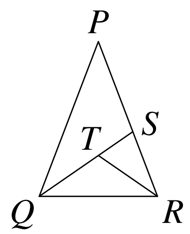
23. Several points were marked on a line. Renard then marked another point between each pair of adjacent points on the line. He performed this process a total of four times. There were then 225 points marked on the line. How many points were marked on the line initially?

A 15 B 16 C 20 D 25 E 30

Solution **A**

Let the original number of points be n. Marking an extra point between each pair of adjacent points would add an extra $n - 1$ points, giving $2n - 1$ points in total after applying the process once. When this process is repeated, there would be $2(2n - 1) - 1 = 4n - 3$ points marked after the second application, $2(4n - 3) - 1 = 8n - 7$ points after the third application and $2(8n - 7) - 1 = 16n - 15$ points after the fourth application. The question tells us that there were 225 points after the fourth application of the process and hence $16n - 15 = 225$, which has solution $n = 15$. Therefore there were 15 points marked on the line initially.

24. An isosceles triangle PQR, in which $PQ = PR$, is split into three separate isosceles triangles, as shown, so that $PS = SQ$, $RT = RS$ and $QT = RT$.
What is the size, in degrees, of angle QPR?



A 24 B 28 C 30 D 35 E 36

Solution **E**

Let the size, in degrees, of angle QPR be x. Since triangle PSQ is isosceles, angle $PQS = x$ and, using the external angle theorem, angle $RST = 2x$. Since triangle STR is isosceles, angle $STR = 2x$ and, since angles on a straight line add to 180°, angle $QTR = 180 - 2x$. Since triangle QTR is isosceles and angles in a triangle add to 180°, angle $TQR = (180 - (180 - 2x))/2 = x$. Therefore angle $PQR = x + x = 2x$ and, since triangle PQR is also isosceles, angle $PRQ = 2x$. Therefore, in triangle PQR, we have $x + 2x + 2x = 180$, since angles in a triangle add to 180°. Hence $x = 36$ and so the size, in degrees, of angle QPR is 36.

25. There are 2022 kangaroos and some koalas living across seven parks. In each park, the number of kangaroos is equal to the total number of koalas in all the other parks. How many koalas live in the seven parks in total?

A 288 B 337 C 576 D 674 E 2022

Solution **B**

Let the number of kangaroos in each of the seven parks be P, Q, R, S, T, U and V with the corresponding number of koalas being p, q, r, s, t, u and v. The question tells us that the number of kangaroos in any park is equal to the sum of the numbers of koalas in the other six parks. Therefore we have

$$\begin{aligned} P &= q + r + s + t + u + v, \\ Q &= p + r + s + t + u + v, \\ R &= p + q + s + t + u + v, \\ S &= p + q + r + t + u + v, \\ T &= p + q + r + s + u + v, \\ U &= p + q + r + s + t + v, \\ V &= p + q + r + s + t + u. \end{aligned}$$

Adding these equations, we obtain

$$P + Q + R + S + T + U + V = 6(p + q + r + s + t + u + v).$$

The total number of kangaroos in the seven parks is 2022. Hence

$$P + Q + R + S + T + U + V = 2022.$$

Therefore

$$2022 = 6(p + q + r + s + t + u + v)$$

and hence the total number of koalas in the seven parks is

$$p + q + r + s + t + u + v = 2022 \div 6 = 337.$$