The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Hamilton Paper

Thursday 19th March 2015

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. The five-digit integer '$a679b$' is a multiple of 72.

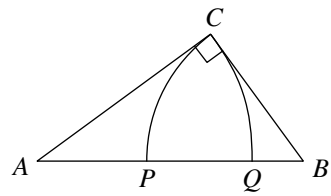
What are the values of a and b?

2. In Vegetable Village it costs 75 pence to buy 2 potatoes, 3 carrots and 2 leeks at the Outdoor Market, whereas it costs 88 pence to buy 2 potatoes, 3 carrots and 2 leeks at the Indoor Market.

To buy a potato, a carrot and a leek costs 6 pence more at the Indoor Market than it does at the Outdoor Market.

What is the difference, in pence, between the cost of buying a carrot at the Outdoor Market and the cost of buying a carrot at the Indoor Market?

3. The diagram shows two circular arcs CP and CQ in a right-angled triangle ABC, where $\angle BCA = 90°$. The centres of the arcs are A and B.



Prove that $\frac{1}{2}PQ^2 = AP \times BQ$.

4. The points A, B and C are the centres of three faces of a cuboid that meet at a vertex. The lengths of the sides of the triangle ABC are 4, 5 and 6.

What is the volume of the cuboid?

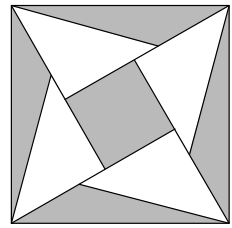
5. Some boys and girls are standing in a row, in some order, about to be photographed. All of them are facing the photographer. Each girl counts the number of boys to her left, and each boy counts the number of girls to his right.

Let the sum of the numbers counted by the girls be G, and the sum of the numbers counted by the boys be B.

Prove that $G = B$.

6. The diagram shows four identical white triangles symmetrically placed on a grey square. Each triangle is isosceles and right-angled.

The total area of the visible grey regions is equal to the total area of the white triangles.



What is the smallest angle in each of the (identical) grey triangles in the diagram?