The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Hamilton Paper

Thursday 10th March 2016

All candidates must be in *School Year 10* (England and Wales), *S3* (Scotland), or *School Year 11* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.**
   Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.
   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.
   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Mathematics, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

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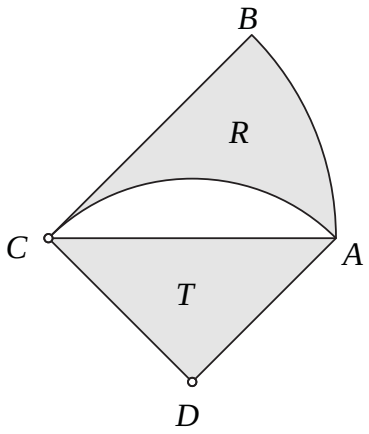
H1. No digit of the positive integer N is prime. However, all the single-digit primes divide N exactly.

What is the smallest such integer N?

H2. The diagram shows two arcs. Arc AB is one eighth of a circle with centre C, and arc AC is one quarter of a circle with centre D. The points A and B are joined by straight lines to C, and A and C are joined by straight lines to D.

Prove that the area of the shaded triangle T is equal to the area of the shaded region R.



H3. Alex is given £1 by his grandfather and decides:

(i) to spend at least one third of the £1 on toffees at 5p each;

(ii) to spend at least one quarter of the £1 on packs of bubblegum at 3p each; and

(iii) to spend at least one tenth of the £1 on jellybeans at 2p each.

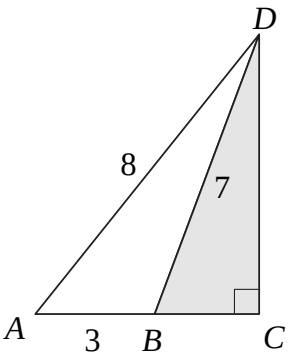
He only decides how to spend the rest of the money when he gets to the shop, but he spends all of the £1 on toffees, packs of bubblegum and jellybeans.

What are the possibilities for the number of jellybeans that he buys?

H4. The diagram shows a right-angled triangle ACD with a point B on the side AC.

The sides of triangle ABD have lengths 3, 7 and 8, as shown.

What is the area of triangle BCD?



H5. James chooses five different positive integers, each at most eight, so that their mean is equal to their median.

In how many different ways can he do this?

H6. Tony multiplies together at least two consecutive positive integers. He obtains the six-digit number N. The left-hand digits of N are ‘47’, and the right-hand digits of N are ‘74’.

What integers does Tony multiply together?