United Kingdom
Mathematics Trust

Intermediate Mathematical Olympiad

Hamilton paper

Thursday 19 March 2020

supported by XTX Markets Overleaf

England & Wales: Year 10
Scotland: S3
Northern Ireland: Year 11

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

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☎ 0113 343 2339 enquiry@ukmt.org.uk www.ukmt.org.uk

- ◇ *Do not hurry, but spend time working carefully on one question before attempting another.*
- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
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- ◇ *Give full written solutions, including mathematical reasons as to why your method is correct.*
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- ◇ *Do* not *hand in rough work.*

1. Arun and Disha have some numbered discs to share out between them. They want to end up with one pile each, not necessarily of the same size, where Arun's pile contains exactly one disc numbered with a multiple of 2 and Disha's pile contains exactly one disc numbered with a multiple of 3. For each case below, either count the number of ways of sharing the discs, or explain why it is impossible to share them in this way.

(a) They start with ten discs numbered from 1 to 10.

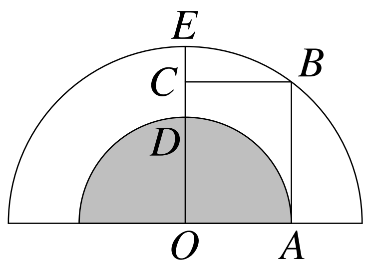
(b) They start with twenty discs numbered from 1 to 20.

2. In the UK, 1p, 2p and 5p coins have thicknesses of 1.6 mm, 2.05 mm and 1.75 mm respectively.

Using only 1p and 5p coins, Joe builds the shortest (non-empty) stack he can whose height in millimetres is equal to its value in pence. Penny does the same but using only 2p and 5p coins.

Whose stack is more valuable?

3. The diagram shows two semicircles with a common centre O and a rectangle $OABC$. The line through O and C meets the small semicircle at D and the large semicircle at E. The lengths CD and CE are equal.

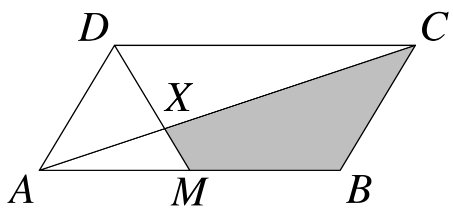


What fraction of the large semicircle is shaded?

4. Piercarlo chooses n integers from 1 to 1000 inclusive. None of his integers is prime, and no two of them share a factor greater than 1.

What is the greatest possible value of n?

5. In the diagram, $ABCD$ is a parallelogram, M is the midpoint of AB and X is the point of intersection of AC and MD.



What is the ratio of the area of $MBCX$ to the area of $ABCD$?

6. We write $\lfloor x \rfloor$ to represent the largest integer less than or equal to x. So, for example, $\lfloor 1.7 \rfloor = 1$, $\lfloor 2 \rfloor = 2$, $\lfloor \pi \rfloor = 3$ and $\lfloor -0.4 \rfloor = -1$.

Find all real values of x such that $\lfloor 3x + 4 \rfloor = \lfloor 5x - 1 \rfloor$.

