United Kingdom
Mathematics Trust

Intermediate Mathematical Olympiad

Hamilton paper

Monday 15 March 2021

supported by [XTX] MARKETS Overleaf

England & Wales: Year 10
Scotland: S3
Northern Ireland: Year 11

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. Write on one side of the paper only and start each question on a fresh sheet.
5. Write your participant ID and question number neatly in the top left corner of each page and arrange them with your cover sheet on top, so that your teacher can easily upload them to the marking platform. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

Enquiries about the Intermediate Mathematical Olympiad should be sent to:

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☎ 0113 365 1121 enquiry@ukmt.org.uk www.ukmt.org.uk

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- ◇ *Try to finish whole questions even if you cannot do many.*
- ◇ *You will have done well if you hand in full solutions to two or more questions.*
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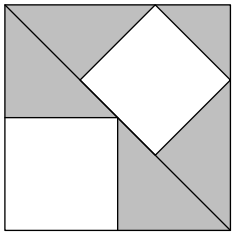
- ◇ *Do* not *hand in rough work.*

1. Naomi has a broken calculator. All it can do is either add one to the previous answer, or square the previous answer. (It performs the operations correctly.) Naomi starts with 2 on the screen. In how many ways can she obtain an answer of 1000?

2. The diagram shows two unshaded squares inside a larger square.

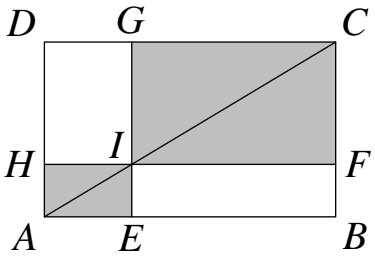


What fraction of the larger square is shaded?

3. For how many positive integers n less than 200 is n^n a cube and $(n+1)^{n+1}$ a square?

4. $ABCD$ is a rectangle with area 6 cm^2.

The point E lies on AB, F lies on BC, G lies on CD and H lies on DA. The point I lies on AC and is the point of intersection of EG and FH, and $AEIH$ and $IFCG$ are both rectangles. One possible diagram is shown to the right.



Given that the combined area of $AEIH$ and $IFCG$ is 4 cm^2, find all possible values for the area of rectangle $AEIH$ in cm^2.

5. Find all real numbers x, y, z such that $x^2 + y^2 + z^2 = x - z = 2$.

6. Humpty buys a box of 15 eggs, with 3 rows and 5 columns. Each meal he removes one egg to cook and eat. If necessary, he moves one or more eggs in the box so that between meals there are always two lines of reflective symmetry. What is the smallest total number of extra egg moves he can make while he empties the box?

Note: You must carefully justify that your answer is minimal; that it is impossible to make fewer extra egg moves while emptying the box.

