United Kingdom
Mathematics Trust

Andrew Jobbings Senior Kangaroo

Thursday 2 December 2021

a member of the Association Kangourou sans Frontières

supported by [XTX] MARKETS Overleaf

England & Wales: Year 13 or below
Scotland: S6 or below
Northern Ireland: Year 14 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the example on the Answer Sheet that shows how to code your answers.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer;
   There is no penalty for giving an incorrect answer.
7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Andrew Jobbings Senior Kangaroo should be sent to:

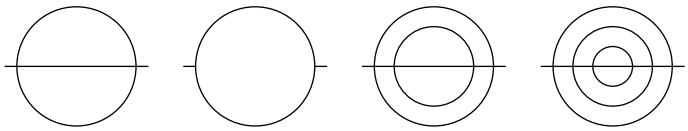
UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT

☎ 0113 365 1121 challenges@ukmt.org.uk www.ukmt.org.uk

1. Adil was born in 2015. His younger sister Bav was born in 2018. What is the minimum number of days by which Adil is older than Bav?

2. The total T is obtained as the sum of the integers from 2006 to 2036 inclusive. What is the sum of all the prime factors of T?

3. How many of the figures shown can be drawn with one continuous line without drawing a segment twice?

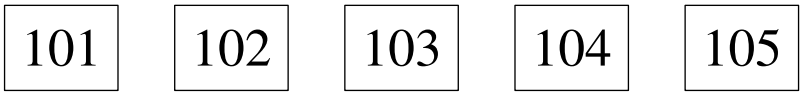


4. On each side of a right-angled triangle, a semicircle is drawn with that side as a diameter. The areas of the three semicircles are x^2, $3x$ and 180 where x^2 and $3x$ are both less than 180. What is the area of the smallest semicircle ?

5. $T = \sqrt{(2021 + 2021) + (2021 - 2021) + (2021 \times 2021) + (2021 \div 2021)}$.
What is the largest prime factor of T?

6. Into how many regions do the x-axis and the graphs of $y = 2 - x^2$ and $y = x^2 - 1$ split the plane?

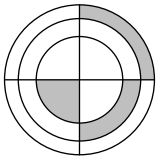
7. Five cards have the numbers 101, 102, 103, 104 and 105 on their fronts.



On the reverse, each card has one of five different positive integers: a, b, c, d and e respectively.
We know that $c = be$, $a + b = d$ and $e - d = a$.
Frankie picks up the card which has the largest integer on its reverse. What number is on the front of Frankie's card?

8. The geometric mean of a set of n positive numbers is defined as the n-th root of the product of those numbers.
Yasmeen writes down a set of four numbers which have a geometric mean of 2048.
Zak writes down a set of four numbers which have a geometric mean of 8.
What is the geometric mean of the combined set of the eight numbers written by Yasmeen and Zak?

9. In the figure shown there are three concentric circles and two perpendicular diameters. The three shaded regions have equal area. The radius of the small circle is 2. The product of the three radii is Y.
What is the value of Y^2?



10. A dealer bought two cars. He sold the first one for 40% more than he paid for it and the second one for 60% more than he paid for it. The total sum he received for the two cars was 54% more than the total sum he paid for them. When written in its lowest terms, the ratio of the prices the dealer paid for the first and the second car was $a : b$. What is the value of $11a + 13b$?

11. Billie has a die with the numbers 1, 2, 3, 4, 5 and 6 on its six faces.
Niles has a die which has the numbers 4, 4, 4, 5, 5 and 5 on its six faces.
When Billie and Niles roll their dice the one with the larger number wins. If the two numbers are equal it is a draw.
The probability that Niles wins, when written as a fraction in its lowest terms, is $\frac{p}{q}$. What is the value of $7p + 11q$?

12. There are 2021 balls in a crate. The balls are numbered from 1 to 2021. Erica works out the digit sum for each ball. For example, the digit sum of 2021 is 5, since $2 + 0 + 2 + 1 = 5$.
Erica notes that balls with equal digit sums have the same colour and balls with different digit sums have different colours.
How many different colours of balls are there in the crate?

13. A multiplication table of the numbers 1 to 10 is shown. What is the sum of all the odd products in the complete table?

×	1	2	3	⋯	10
1	1	2	3	⋯	10
2	2	4	6	⋯	20
⋮	⋮				⋮
10	10	20	30	⋯	100

14. The graph of $(x^2 + y^2 - 2x)^2 = 2(x^2 + y^2)^2$ meets the x-axis in p different places and meets the y-axis in q different places.
What is the value of $100p + 100q$?

15. Which is the lowest numbered statement which is true?
Statement 201: "Statement 203 is true".
Statement 202: "Statement 201 is true".
Statement 203: "Statement 206 is false".
Statement 204: "Statement 202 is false".
Statement 205: "None of the statements 201, 202, 203 or 204 are true".
Statement 206: "1 + 1 = 2".

16. A polygon is said to be *friendly* if it is regular and it also has angles that when measured in degrees are either integers or *half-integers* (i.e. have a decimal part of exactly 0.5). How many different friendly polygons are there?

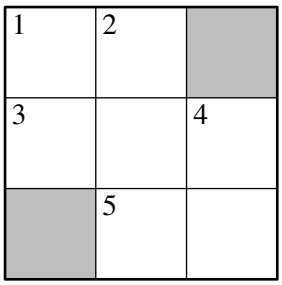
17. Find the value of R, given that the numbers Q and R are defined as:

$Q = 202^1 + 20^{21} + 2^{021}$;
R is the remainder when Q is divided by 1000.

18. Some years, like 2022, have the property that they use at most two different digits. How many years between 1 and 9999, inclusive, have this property?

19. The function $f(x)$ is defined as $f(x) = \frac{x-1}{x+1}$.
The equation $f(x^2) \times f(x) = 0.72$ has two solutions a and b, where $a > b$.
What is the value of $19a + 7b$?

20. Each cell in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are satisfied. The digits used are not necessarily distinct.
What is the answer to 2 DOWN?



ACROSS

1. A prime which is the sum of two squares
3. Twice the answer to 2 DOWN

DOWN

1. $p \times q$, where p, q are prime and $q = p + 4$
4. 60% of 5 ACROSS

