Topic Test: OxfordAQA International A level Physics

Gravitational fields and satellites

Name: ______________________

Class: ______________________

Date: ______________________

Time: **54 minutes**

Marks: **42 marks**

Comments:

1 (a) State Newton's law of gravitation.

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(2)

(b) In 1798 Cavendish investigated Newton's law by measuring the gravitational force between two unequal uniform lead spheres. The radius of the larger sphere was 100 mm and that of the smaller sphere was 25 mm.

(i) The mass of the smaller sphere was 0.74 kg. Show that the mass of the larger sphere was about 47 kg.

density of lead = 11.3×10^3 kg m^{-3}

(2)

(ii) Calculate the gravitational force between the spheres when their surfaces were in contact.

answer = ______________________ N

(2)

(c) Modifications, such as increasing the size of each sphere to produce a greater force between them, were considered in order to improve the accuracy of Cavendish's experiment. Describe and explain the effect on the calculations in part (b) of doubling the radius of both spheres.

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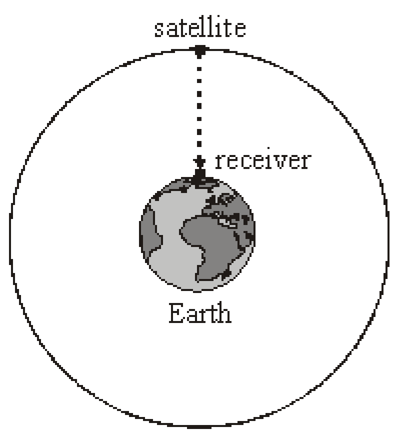
(4)
(Total 10 marks)

2 The Global Positioning System (GPS) is a system of satellites that transmit radio signals which can be used to locate the position of a receiver anywhere on Earth.



(a) A receiver at sea level detects a signal from a satellite in a circular orbit when it is passing directly overhead as shown in the diagram above.

(i) The microwave signal is received 68 ms after it was transmitted from the satellite. Calculate the height of the satellite.

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(ii) Show that the gravitational field strength of the Earth at the position of the satellite is 0.56 N kg^{-1}.

mass of the Earth = 6.0×10^{24} kg
mean radius of the Earth = 6400 km

(4)

(b) For the satellite in this orbit, calculate

(i) its speed,

(ii) its time period.

(5)
(Total 9 marks)

3 A space mission is planned to remove a small boulder of mass 8.2×10^3 kg from the surface of an asteroid.

(a) The gravitational field strength is 1.77×10^{-4} N kg^{-1} at the surface of the asteroid.

Calculate the weight of the boulder at the surface of the asteroid.

weight = ____________________ N

(1)

(b) Explain why gravitational potential is always a negative quantity.

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(2)

(c) The gravitational potential at the surface of the asteroid, due to the asteroid's gravitational field, is -3.99×10^{-2} J kg^{-1}
A spacecraft of mass 1.8×10^4 kg removes the boulder to a position where the gravitational potential is negligible.

Calculate the work done against the asteroid's gravitational field.

work done = ____________________ J

(1)

(d) The boulder will be placed into a circular orbit around the Moon.

Show that the period T of the circular orbit of the boulder around the Moon is given by

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

where r = the radius of the orbit
and M = mass of the Moon.

(3)

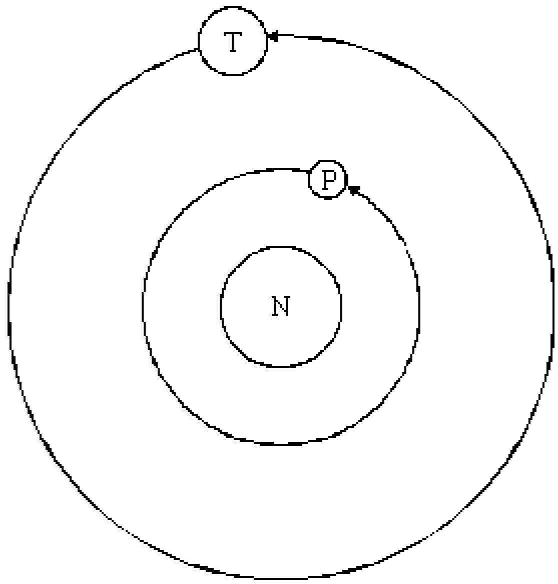
(e) The orbital period of the orbit around the Moon is 24 h.
The mass of the Moon is 7.35×10^{22} kg.

Calculate the radius of the orbit.

radius of orbit = ____________________ m

(2)
(Total 9 marks)

4 The diagram below (not to scale) shows the planet Neptune (N) with its two largest moons, Triton (**T**) and Proteus (**P**). Triton has an orbital radius of 3.55×10^8 m and that of Proteus is 1.18×10^8 m. The orbits are assumed to be circular.



(a) Explain why the velocity of each moon varies whilst its orbital speed remains constant.

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(1)

(b) Write down an equation that shows how Neptune's gravitational attraction provides the centripetal force required to hold Triton in its orbit. Hence show that it is unnecessary to know the mass of Triton in order to find its angular speed.

(3)

(c) Show that $\frac{\text{the orbital period of Triton}}{\text{the orbital period of Proteus}}$ is approximately 5.2.

(4)
(Total 8 marks)

5 What is a unit for G, the gravitational constant?

A kg m^3 s^{-2} ⬭

B kg^{-1} m^3 s^2 ⬭

C J m^{-1} kg^2 ⬭

D J m kg^{-2} ⬭

(Total 1 mark)

6 A spacecraft of mass m is at the mid-point between the centres of a planet of mass M_1 and its moon of mass M_2. If the distance between the spacecraft and the centre of the planet is d, what is the magnitude of the resultant gravitational force on the spacecraft?

A $\dfrac{Gm(M_1 - M_2)}{d}$

B $\dfrac{Gm(M_1 + M_2)}{d^2}$

C $\dfrac{Gm(M_1 - M_2)}{d^2}$

D $\dfrac{Gm(M_1 + M_2)}{d}$

(Total 1 mark)

7 Gravitational field lines and lines of equipotential

A are straight in a radial field.

B are curved in a radial field.

C intersect at 90° only in uniform fields.

D intersect at 90° in uniform and radial fields.

(Total 1 mark)

8 The diameter of the Earth is 3 times the diameter of planet **P**.
The mass of the Earth is 18 times the mass of **P**.
The acceleration due to gravity at the surface of the Earth is g.

What is the acceleration due to gravity at the surface of **P**?

A $\dfrac{g}{6}$

B $\dfrac{g}{2}$

C $2g$

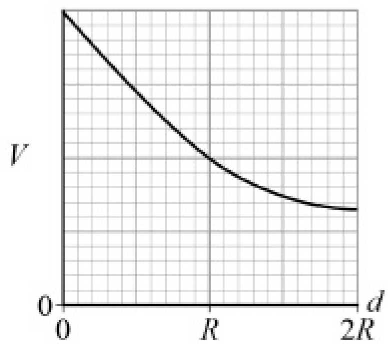
D $6g$

(Total 1 mark)

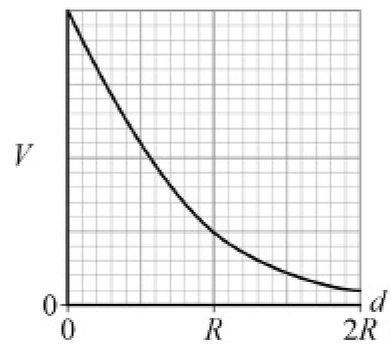
9 In the graphs below, R is the radius of the Earth.

Which graph shows the variation of gravitational potential V with distance d from the surface of the Earth?

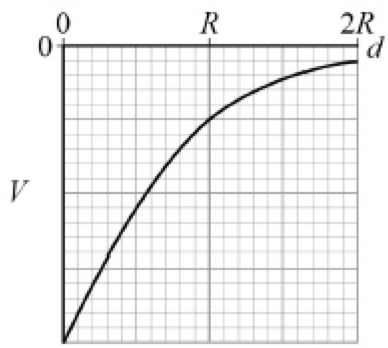
A



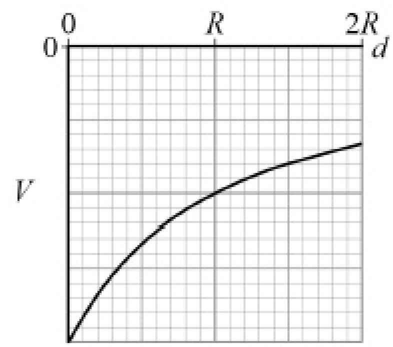
B



C



D



A ○

B ○

C ○

D ○

(Total 1 mark)

10 Ganymede and Io are two moons of Jupiter. The orbital radius of Ganymede is greater than the orbital radius of Io.

Which row shows the moon with the greater orbital speed and the moon with the greater angular speed?

	Greater orbital speed	Greater angular speed	
A	Io	Io	⬭
B	Io	Ganymede	⬭
C	Ganymede	Io	⬭
D	Ganymede	Ganymede	⬭

(Total 1 mark)

Mark schemes

1 (a) force of attraction between two point masses (or particles) **(1)**

proportional to product of masses **(1)**

inversely proportional to square of distance between them **(1)**

[**alternatively**

quoting an equation, $F = \frac{GM_1M_2}{r^2}$ with all terms defined **(1)**

reference to point masses (or particles) **or** *r* is distance between centres **(1)**

F identified as an attractive force **(1)**]

max 2

(b) (i) mass of larger sphere M_L (= $\frac{4}{3}\pi r^3\rho$) = $\frac{4}{3}$ π × $(0.100)^3$ × 11.3 × 10^3 **(1)**

= 47(.3) (kg) **(1)**

[**alternatively**

use of *M* μ r^3 gives $\frac{M_L}{0.74} = \left(\frac{100}{25}\right)^3$ **(1)** (= 64)

and M_L = 64 × 0.74 = 47(.4) (kg) **(1)**]

2

(ii) gravitational force F $\left(= \frac{GM_L M_S}{x^2}\right) = \frac{6.67 \times 10^{-11} \times 47.3 \times 0.74}{0.125^2}$ **(1)**

= 1.5 × 10^{-7} (N) **(1)**

2

(c) for the spheres, mass μ volume (or μ r^3, or $M = \frac{4}{3}\pi r^3\rho$) **(1)**

mass of either sphere would be 8 × greater (378 kg, 5.91 kg) **(1)**

this would make the force 64 × greater **(1)**

but separation would be doubled causing force to be 4 × smaller **(1)**

net effect would be to make the force (64/4) = 16 × greater **(1)**
(ie 2.38 × 10^{-6} N)

max 4

[10]

2 (a) (i) h (= ct) (= $3.0 \times 10^8 \times 68 \times 10^{-3}$) = $2.0(4) \times 10^7$ m **(1)**

(ii) $g = (-)\frac{GM}{r^2}$ **(1)**

r (= $6.4 \times 10^6 + 2.04 \times 10^7$) = 2.68×10^7 (m) **(1)**

(allow C.E. for value of h from (i) for first two marks, but not 3rd)

$g = \frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(2.68 \times 10^7)^2}$ **(1)** (= 0.56 N kg^{-1})

4

(b) (i) $g = \frac{v^2}{r}$ **(1)**

$v = [0.56 \times (2.68 \times 10^7)]^{½}$ **(1)**

= 3.9×10^3m s^{-1} **(1)** (3.87×10^3 m s^{-1})

(allow C.E. for value of r from a**(**ii**)**

[or $v^2 = \frac{GM}{r}$ = **(1)**

$v = \left(\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{2.68 \times 10^7}\right)^{1/2}$ **(1)**

= 3.9×10^3 m s^{-1} **(1)**]

(ii) $T\left(= \frac{2\pi r}{v}\right) = \frac{2\pi \times 2.68 \times 10^7}{3.87 \times 10^3}$ **(1)**

= $4.3(5) \times 10^4$s **(1)** (12.**(1)** hours)

(use of $v = 3.9 \times 10^3$ gives T = 4.3**(1)** × 10^4 s = 12.0 hours)

(allow C.E. for value of v from (I)

[alternative for (b):

(i) $v\left(\frac{2\pi r}{T}\right) = \frac{2\pi \times 2.68 \times 10^7}{4.36 \times 10^4}$ **(1)**

$= 3.8(6) \times 10^3$ m s^{-1} **(1)**]

(allow C.E. for value of *r* from (a)(ii) and value of *T*)

(ii) $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ **(1)**

$\left(= \frac{4\pi^2}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}} \times (2.68 \times 10^7)^3\right)$ = (1.90 × 10^9 **(**s^2**) (1)**

$T = 4.3(6) \times 10^4$ s **(1)**

5

[9]

3 (a) 1.45 N to <u>2 or 3 sf</u> **cao**

1

(b) Potential is zero at infinity ✓

Potential is lower (than zero) (at the point in the field) since work needs to be done to move (unit) mass from the point to infinity (against gravitational attraction) wtte ✓

2

(c) 1050 or 1045 J **cao**

Accept 1000 (J) if it is stated to be 2 sf

1

(d) Equates $\frac{mv^2}{r}$ or $mr\omega^2$ with $\frac{GMm}{r^2}$ ✓

Uses $T = \frac{2\pi}{\omega}$ ✓

Clear and convincing manipulation ✓

3

(e) Correct rearrangement or substitution ✓

9.75×10^6 (m) ✓

Expect to see $24 \times 3600 = \sqrt{\dfrac{4\pi^2 r^3}{6.67 \times 10^{-11} 7.35 \times 10^{22}}}$

Or $r = \sqrt[3]{\dfrac{GMT^2}{4\pi^2}}$ *for the 1st mark*

Condone wrong value of t in the substitution for the 1st mark

2

[9]

4 (a) direction changing, velocity vector

B1

1

(b) Newton's law equation

M1

centripetal force equation

M1

cancel mass of Triton

A1

3

(c) $\omega = 2\pi f$ or $\omega = 2\pi/T$

M1

$\omega^2 r^3$ = constant or $\omega^2 = \dfrac{GM}{r^3}$

M1

$\dfrac{T_T^2}{T_P^2} = \dfrac{r_T^3}{r_P^3}$ **or** statement of Kepler III for B3

$\dfrac{T_T}{T_P} = \sqrt{\dfrac{(3.55 \times 10^8)^3}{(1.18 \times 10^8)^3}} = 5.2(2)$

M1

4

[8]

5 D

[1]

6 C

[1]

7 D

[1]

8 B

[1]

9 D

[1]

10 A

[1]