Question	Scheme	Marks	AOs
1(a)	$R=\sqrt{5}$	B1	1.1b
	$\tan\alpha=\frac{1}{2}$ or $\sin\alpha=\frac{1}{\sqrt{5}}$ or $\cos\alpha=\frac{2}{\sqrt{5}}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=0.464$	A1	1.1b
		(3)	
(b)(i)	$3+2\sqrt{5}$	B1ft	3.4
(ii)	$\cos(0.5t+0.464)=1\Rightarrow 0.5t+0.464=2\pi$ $\Rightarrow t=...$	M1	3.4
	$t=11.6$	A1	1.1b
		(3)	
(c)	$3+2\sqrt{5}\cos(0.5t+0.464)=0$ $\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}$	M1	3.4
	$\cos(0.5t+0.464)=-\frac{3}{2\sqrt{5}}\Rightarrow 0.5t+0.464=\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)$ $\Rightarrow t=2\left(\cos^{-1}\left(-\frac{3}{2\sqrt{5}}\right)-0.464\right)$	dM1	1.1b
	So the time required is e.g.: $2(3.977...-0.464)-2(2.306...-0.464)$	dM1	3.1b
	= 3.34	A1	1.1b
		(4)	
(d)	e.g. the "3" would need to vary	B1	3.5c
		(1)	
			(11 marks)

Notes

(a)

B1: $R=\sqrt{5}$ only.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{1}{2}$ or $\sin\alpha=\pm\frac{1}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

It is implied by either awrt 0.464 (radians) or awrt 26.6 (degrees)

A1: $\alpha=$awrt 0.464

(b)(i)

B1ft: For $\left(3+2\sqrt{5}\right)$m or awrt 7.47 m and remember to isw. Condone lack of units.

Follow through on their R value so allow 3 + 2×Their R. (Allow in decimals with at least 3sf accuracy)

(b)(ii)

M1: Uses $0.5t\pm"0.464"=2\pi$ to obtain a value for t

Follow through on their 0.464 but this angle must be in radians.

It is possible in degrees but only using $0.5t\pm"26.6"=360$

A1: Awrt 11.6

Alternative for (b):

$$H = 3 + 4\cos(0.5t) - 2\sin(0.5t) \Rightarrow \frac{\mathrm{d}H}{\mathrm{d}t} = -2\sin(0.5t) - \cos(0.5t) = 0$$

$$\Rightarrow \tan(0.5t) = -\frac{1}{2} \Rightarrow 0.5t = 2.677..., 5.819... \Rightarrow t = 5.36,\ 11.6$$

$$t = 11.6 \Rightarrow H = 7.47$$

Score as follows:

M1: For a complete method:

Attempts $\frac{\mathrm{d}H}{\mathrm{d}t}$ and attempts to solve $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$ for t

A1: For t = awrt 11.6

B1ft: For awrt 7.47 or 3 + 2×Their R

(c)

M1: Uses the model and sets $3 + 2"\sqrt{5}"\cos(...) = 0$ and proceeds to $\cos(...) = k$ where $|k| < 1$.

Allow e.g. $3 + 2"\sqrt{5}"\cos(...) < 0$

dM1: Solves $\cos(0.5t \pm "0.464") = k$ where $|k| < 1$ to obtain at least one value for t

This requires e.g. $2\left(\pi + \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$ or e.g. $2\left(\pi - \cos^{-1}(k) \pm \tan^{-1}\left(\frac{1}{2}\right)\right)$

Depends on the previous method mark.

dM1: A fully correct strategy to find the required duration. E.g. finds 2 consecutive values of t when H = 0 and subtracts. Alternatively finds t when H is minimum and uses the times found correctly to find the required duration.

Depends on the previous method mark.

Examples:

Second time at water level – first time at water level:

$$2\left(\pi + \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right) = 7.02685... - 3.68492...$$

2×(first time at minimum point – first time at water level):

$$2\left(2\left(\pi - \tan^{-1}\left(\frac{1}{2}\right)\right) - 2\left(\pi - \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right) - \tan^{-1}\left(\frac{1}{2}\right)\right)\right) = 2(5.35589... - 3.68492...)$$

Note that both of these examples equate to $4\cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ which is not immediately obvious but may be seen as an overall method.

There may be other methods – if you are not sure if they deserve credit send to review.

A1: Correct value. Must be 3.34 (**not** awrt).

Special Cases in (c):

Note that if candidates have an incorrect α and have e.g. $3 + 2\sqrt{5}\cos(0.5t - 0.464)$, this has no impact on the final answer. So for candidates using $3 + 2\sqrt{5}\cos(0.5t \pm \alpha)$ in (c) allow all the marks including the A mark as a correct method should always lead to 3.34

Some values to look for:

$$0.5t \pm "0.464" = \pm 2.306,\ \pm 3.977,\ \pm 8.598,\ \pm 10.26$$

(d)

B1: Correct refinement e.g. As in scheme. If they suggest a specific function to replace the "3" then it must be sensible e.g. a trigonometric function rather than e.g. a quadratic/linear one.

Question	Scheme	Marks	AOs
2(a)	Deduces that $A = \pm 50$ **or** $b = \frac{1}{4}$	B1	3.4
	Deduces that $A = \pm 50$ **and** $b = \frac{1}{4}$	B1	3.4
	Uses $t = 0, H = 1 \Rightarrow \alpha = ...$ E.g. $1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$	M1	3.4
	$H = \left\| \pm 50\sin\left(\frac{1}{4}t + 1.15\right)° \right\|$	A1	3.3
		(4)	
(b)	E.g. the minimum height above the ground of the passenger on the original model was 0 m or Adding "*d*" means the passenger does not touch the ground.	B1	3.5b
		(1)	
			(5 marks)
Notes:			

(a) Note that B0B1 is not possible

B1: Uses the equation of the given model to deduce that $A = \pm 50$ **or** $b = \frac{1}{4}$ o.e.

May be seen embedded within their equation.

B1: Uses the equation of the given model to deduce that $A = \pm 50$ **and** $b = \frac{1}{4}$ o.e.

May be seen embedded within their equation.

M1: Uses $t = 0$ and $H = 1$ in the equation of the model to find a value for α.

Follow through on their value for *A*. Allow for $\pm 1 = "50"\sin(\alpha)° \Rightarrow \alpha = ...$ where α is in degrees or radians.

Note that in radians $\sin^{-1}\left(\frac{1}{50}\right) \approx \frac{1}{50}$ (0.0200…) which may appear incorrect but is in fact ok.

Also in degrees a value of e.g. 1.14 (truncated) would indicate the method.

A1: Writes down the correct full equation of the model: $H = \left| "\pm" 50\sin\left(\frac{1}{4}t + 1.15\right)° \right|$ o.e.

Condone omission of degrees symbol and allow awrt 1.15 for α.

Allow if a correct equation is seen anywhere in their solution.

(b)

B1: Gives a suitable explanation with no contradictory statements.

Condone "so that pod/capsule/seat/passenger/ferris wheel/it etc. will not hit/touch the ground"

Responses that focus on the starting point of the model are likely to score B0

Question	Scheme	Marks	AOs
3(a)	$a = 60$	B1	3.1b
	$2 = "60" - b(-20)^2 \Rightarrow b = ...$	M1	3.4
	$H = 60 - 0.145(t-20)^2$	A1	3.3
		(3)	
(b)	Height = 2 m	B1	3.4
		(1)	
(c)	$\alpha = 180$ or $\beta = 31$	M1	3.4
	$H = 29\cos(9t + 180)° + 31$	A1	3.3
		(2)	
(d)	e.g. "The model allows for more than one circuit"	B1	3.5a
		(1)	
			(7 marks)

Notes

(a)

B1: $a = 60$ (may be seen in their final equation of the model or implied by 60 substituted for *a* in the model)

M1: Attempts to find *b* by substituting in $t = 0$, $H = 2$ and their *a* and proceeding to a value for *b*.
May be seen as two simultaneous equations formed:
$2 = a - b(-20)^2$ and $60 = a - b(20-20)^2$ proceeding to a value for *b*

A1: $H = 60 - 0.145(t-20)^2$ or equivalent such as $H = -\frac{29}{200}t^2 + 5.8t + 2$ or $H = 60 - \frac{29}{200}(t-20)^2$ isw once a correct equation for the model is seen. Must be in terms of *H* and *t*. If they just state $a = 60$, $b = 0.145$ then A0
A correct answer with no working seen scores full marks.

(b)

B1: 2 cao (condone lack of units) This can be scored even if their model in (a) is incorrect (they may have used symmetry to determine this value)

(c)

M1: $(\alpha =)\ 180$ or $(\beta =)\ 31$ Condone $(\alpha =)\ \pi$

A1: $H = 29\cos(9t + 180)° + 31$ or equivalent e.g. $H = -29\cos(9t) + 31$ isw once a correct equation for the model is seen. Must be in terms of *H* and *t*. If they just state $\alpha = 180$, $\beta = 31$ then A0.
A correct equation with no working seen scores both marks. Does not require the degree symbol.

(d)

B1: Score for a reason which makes reference to any of

- the alternative model allows repetition (allow phrases e.g. "multiple cycles", "repeated circuits", "cyclical", "periodic", "loops around", "the original model can only go up and down once")
- the alternative model after 2 minutes the carriage will be back at the start (e.g."at 2 mins, $H = 2$")
- the original/quadratic model after 40 seconds (or any time after this) will be negative (e.g. "the height will be negative which cannot happen")
- the original model after 2 minutes would not be back at the start

Do not allow vague responses on their own e.g. "the original model is a parabola"
If calculations are used then they must be correct using a correct model (allow rounded or truncated)
Look for a valid reason and ignore reference to anything else as long as it does not contradict

t	0	5	10	15	20	25	30	35	40	45	50	55	60	80	100	120
h	2	27	46	56	60	56	46	27	2	-31	-71	-118	-172	-462	-868	-1390