1. A parallelogram $PQRS$ has area $50\,\text{cm}^2$

Given

- PQ has length $14\,\text{cm}$
- QR has length $7\,\text{cm}$
- angle SPQ is obtuse

find

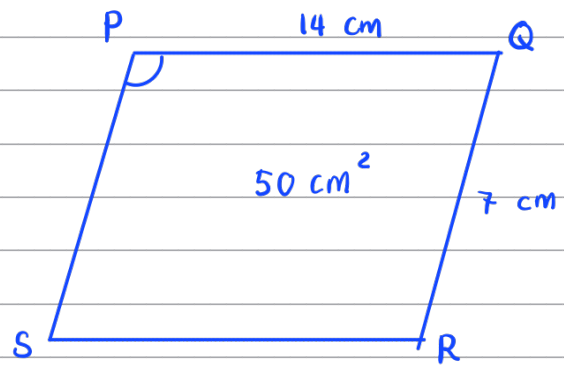
(a) the size of angle SPQ, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal SQ, in cm, to one decimal place.

(2)

a)



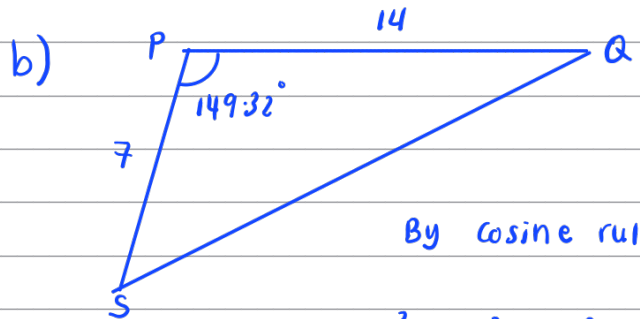
$50 = 7 \times 14 \sin(\angle SPQ)$ (1)

$\Rightarrow \angle SPQ = 180 - \arcsin\left(\frac{50}{98}\right)$ (1)

$\angle SPQ = 149.32°$ (1)

$\angle SPQ$ is obtuse. Since $\arcsin\left(\frac{50}{98}\right)$ gives us acute angle, we substract $180°$ with the solution of $\arcsin\left(\frac{50}{98}\right)$.

b)



By cosine rule:

$SQ^2 = 7^2 + 14^2 - 2(7)(14)\cos(149.32°)$ (1)

$SQ^2 = 413.57046$

$SQ = 20.3\ \text{cm}$ (1)

2.

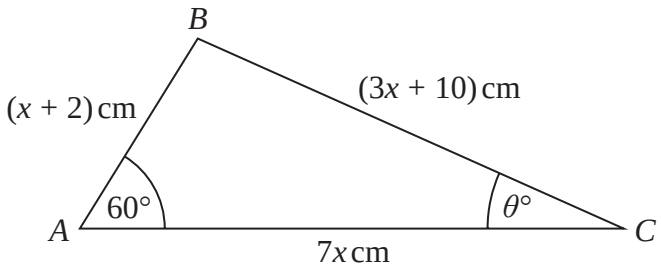


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

a) (i) To get an equation, we can use cosine rule since we have one angle with all 3 sides.

$BC^2 = AB^2 + AC^2 - 2 \times AB \times AC \times \cos \angle BAC$

$(3x+10)^2 = (x+2)^2 + (7x)^2 - 2(x+2)(7x)\cos 60°$ (1)

$9x^2 + 60x + 100 = x^2 + 4x + 4 + 49x^2 - 7x^2 - 14x$ (1)

$9x^2 + 60x + 100 = 43x^2 - 10x + 4$

$34x^2 - 70x - 96 = 0$

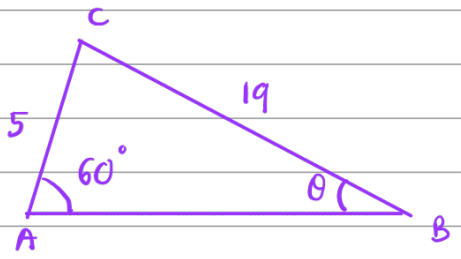
$\therefore 17x^2 - 35x - 48 = 0$ (1)

(ii) $17x^2 - 35x - 48 = (17x + 16)(x - 3) = 0$

$x = \frac{-16}{17}$, $x = 3$ (1) length cannot be negative value

$\therefore$ Since x can only be positive, $x = 3$ is the only solution.

b) when $x = 3$,



$AB = (x+2)\text{ cm} = 5\text{ cm}$

$BC = (3x+10)\text{ cm} = 19\text{ cm}$

using sin rule to get the angle θ :

$$\frac{\sin\theta}{5\text{ cm}} = \frac{\sin 60^\circ}{19\text{ cm}} \quad (1)$$

$$\sin\theta = \frac{5}{19}\sin 60^\circ$$

$$\theta = \sin^{-1}\frac{5\sqrt{3}}{38} \quad (1)$$

$$= 13.17^\circ \approx 13.2^\circ \text{ (1 d.p.)} \quad (1)$$

3.

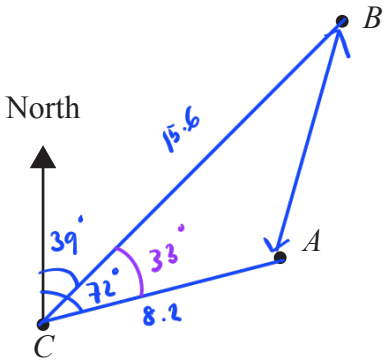


Figure 1

Figure 1 is a sketch showing the position of three phone masts, A, B and C.

The masts are identical and their bases are assumed to lie in the same horizontal plane.

From mast C

- mast A is 8.2 km away on a bearing of 072°
- mast B is 15.6 km away on a bearing of 039°

(a) Find the distance between masts A and B, giving your answer in km to one decimal place.

(3)

An engineer needs to travel from mast A to mast B.

(b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels.

(1)

a) $\angle BAC$ = bearing A - bearing B

= $72° - 39° = 33°$ (1)

Cosine rule :

$A^2 = B^2 + C^2 - 2BC \cos A$

Using cosine rule to get the length AB :

$AB^2 = 15.6^2 + 8.2^2 - 2(15.6)(8.2) \cos 33°$ (1)

$AB^2 = 96.03$

$AB = 9.8$ km (1)

Question 3 continued

b) In a real case scenario, the road is unlikely to be completely straight. Therefore, the distance AB is likely to be longer. (1)

(Total for Question 3 is 4 marks)