Question	Scheme	Marks	AOs
1(a)	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\cos 2\theta+\sin 2\theta}$ or $\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\cos 2\theta+\sin 2\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	M1	2.1
	$\frac{1-\cos 2\theta+\sin 2\theta}{1+\cos 2\theta+\sin 2\theta}=\frac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$	A1	1.1b
	$=\frac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}=\frac{2\sin\theta\left(\sin\theta+\cos\theta\right)}{2\cos\theta\left(\cos\theta+\sin\theta\right)}$	dM1	2.1
	$=\frac{\sin\theta}{\cos\theta}=\tan\theta$ *	A1*	1.1b
		(4)	
(b)	$\frac{1-\cos 4x+\sin 4x}{1+\cos 4x+\sin 4x}=3\sin 2x\Rightarrow\tan 2x=3\sin 2x$ o.e	M1	3.1a
	$\Rightarrow\sin 2x-3\sin 2x\cos 2x=0$ $\Rightarrow\sin 2x\left(1-3\cos 2x\right)=0$ $\Rightarrow\left(\sin 2x=0,\right)\ \cos 2x=\frac{1}{3}$	A1	1.1b
	$x=90°$, awrt $35.3°$, awrt $144.7°$	A1 A1	1.1b 2.1
		(4)	
			(8 marks)

Notes

(a)

M1: Attempts to use a correct double angle formulae for both $\sin 2\theta$ and $\cos 2\theta$ (seen once).
The application of the formula for $\cos 2\theta$ must be the one that cancels out the "1"
So look for $\cos 2\theta = 1 - 2\sin^2\theta$ in the numerator or $\cos 2\theta = 2\cos^2\theta - 1$ in the denominator
Note that $\cos 2\theta = \cos^2\theta - \sin^2\theta$ may be used as well as using $\cos^2\theta + \sin^2\theta = 1$

A1: $\dfrac{1-\left(1-2\sin^2\theta\right)+2\sin\theta\cos\theta}{1+\left(2\cos^2\theta-1\right)+2\sin\theta\cos\theta}$ or $\dfrac{2\sin^2\theta+2\sin\theta\cos\theta}{2\cos^2\theta+2\sin\theta\cos\theta}$

dM1: Factorises numerator and denominator in order to demonstrate cancelling of ($\sin\theta + \cos\theta$)

A1*: Fully correct proof with no errors.

You must see an intermediate line of $\dfrac{2\sin\theta\left(\cancel{\sin\theta+\cos\theta}\right)}{2\cos\theta\left(\cancel{\cos\theta+\sin\theta}\right)}$ or $\dfrac{\sin\theta}{\cos\theta}$ or even $\dfrac{2\sin\theta}{2\cos\theta}$

Withhold this mark if you see, within the body of the proof,

- notational errors. E.g. $\cos 2\theta = 1 - 2\sin^2$ or $\cos\theta^2$ for $\cos^2\theta$
- mixed variables. E.g. $\cos 2\theta = 2\cos^2 x - 1$

(b)

M1: Makes the connection with part (a) and writes the lhs as $\tan 2x$. Condone $x \leftrightarrow \theta$ $\tan 2\theta = 3\sin 2\theta$

A1: Obtains $\cos 2x = \dfrac{1}{3}$ o.e. with $x \leftrightarrow \theta$. You may see $\sin^2 x = \dfrac{1}{3}$ or $\cos^2 x = \dfrac{2}{3}$ after use of double angle formulae.

A1: Two "correct" values. Condone accuracy of awrt $90°,\ 35°,\ 145°$
Also condone radian values here. Look for 2 of awrt 0.62, 1.57, 2.53

A1: All correct (allow awrt) and no other values in range. Condone $x \leftrightarrow \theta$ if used consistently

....

Answers without working in (b): Just answers and no working score 0 marks.
If the first line is written out, i.e. $\tan 2x = 3\sin 2x$ followed by all three correct answers score 1100 .

Question	Scheme	Marks	AOs
2(a)	Attempts to use both $\sin(x-60°)=\pm\sin x\cos 60°\pm\cos x\sin 60°$ $\cos(x-30°)=\pm\cos x\cos 30°\pm\sin x\sin 30°$	M1	2.1
	Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$	A1	1.1b
	Either uses $\frac{\sin x}{\cos x}=\tan x$ and attempts to make $\tan x$ the subject E.g. $(2\cos 60°-\sin 30°)\tan x=\cos 30°+2\sin 60°$ Or attempts $\sin 30°$ etc with at least two correct and collects terms in $\sin x$ and $\cos x$ E.g. $\left(2\times\frac{1}{2}-\frac{1}{2}\right)\sin x=\left(2\times\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\right)\cos x$,	M1	2.1
	Proceeds to given answer showing all key steps E.g. $\frac{1}{2}\tan x=\frac{3\sqrt{3}}{2}\Rightarrow\tan x=3\sqrt{3}$ *	A1*	1.1b
		(4)	
(b)	Deduces that $x=2\theta+60°$	B1	2.2a
	$\tan(2\theta+60°)=3\sqrt{3}\Rightarrow 2\theta+60°=79.1°, 259.1°, ...$	M1	1.1b
	Correct method to find one value of θ E.g $\theta=\frac{79.1°-60°}{2}$	dM1	1.1b
	$\theta=$ awrt $9.6°, 99.6°$ (See note)	A1	2.1
		(4)	
			(8 marks)
Notes:			

(a)

M1: Attempts to use both compound angle expansions to set up an equation in sin x and cos x
The terms must be correct but condone sign errors and a slip on the multiplication of 2

A1: Correct equation $2\sin x\cos 60°-2\cos x\sin 60°=\cos x\cos 30°+\sin x\sin 30°$ o.e.
Note that $\cos 60°=\sin 30°$ and $\cos 30°=\sin 60°$
Also allow this mark for candidates who substitute in their trigonometric values "early"

$$2\sin x\times\frac{1}{2}-2\cos x\times\frac{\sqrt{3}}{2}=\cos x\times\frac{\sqrt{3}}{2}+\sin x\times\frac{1}{2} \text{ o.e.}$$

M1: Shows the necessary progress towards showing the given result.
There are three key moves, two of which must be shown for this mark.

- uses $\frac{\sin x}{\cos x}=\tan x$ to form an equation in just tan x.
- uses exact numerical values for $\sin 30°, \sin 60°, \cos 30°, \cos 60°$ with at least two correct
- collects terms in $\sin x$ and $\cos x$ or alternatively in $\tan x$

A1*: Proceeds to the given answer with accurate work showing all necessary lines.

Examples of two proofs showing all necessary lines

E.g. I $2\sin x\cos 60° - 2\cos x\sin 60° = \cos x\cos 30° + \sin x\sin 30°$

$\sin x\left(2\cos 60° - \sin 30°\right) = \cos x\left(\cos 30° + 2\sin 60°\right)$ 1. collect terms

$\left(2\cos 60° - \sin 30°\right)\tan x = \cos 30° + 2\sin 60°$ 2. $\frac{\sin x}{\cos x} = \tan x$ so M1

$\tan x = \frac{\cos 30° + 2\sin 60°}{2\cos 60° - \sin 30°} = \frac{\frac{\sqrt{3}}{2} + \sqrt{3}}{1 - \frac{1}{2}} = 3\sqrt{3}$ 3..uses values and completes proof A1*

E.g II

$2\sin x \times \frac{1}{2} - 2\cos x \times \frac{\sqrt{3}}{2} = \cos x \times \frac{\sqrt{3}}{2} + \sin x \times \frac{1}{2}$ 1.uses values

$\Rightarrow \frac{1}{2}\sin x = \frac{3\sqrt{3}}{2}\cos x$ 2.collects terms so M1

$\Rightarrow \tan x = 3\sqrt{3}$

3. $\frac{\sin x}{\cos x} = \tan x$ completes proof A1*

(b) Hence

B1: Deduces that $x = 2\theta + 60°$ o.e such as $\theta = \frac{x - 60°}{2}$

This is implied for sight of the equation $\tan\left(2\theta + 60°\right) = 3\sqrt{3}$

M1: Proceeds from $\tan\left(2\theta \pm \alpha°\right) = 3\sqrt{3} \Rightarrow 2\theta \pm \alpha° =$ one of $79.1°, 259.1°, ...$ where $\alpha \neq 0$

One angle for $\arctan\left(3\sqrt{3}\right)$ **must** be correct in degrees or radians(3sf). FYI radian answers 1.38, 4.52

dM1: Correct method to find one value of θ from their $2\theta \pm \alpha° = 79.1°$ to $\theta = \frac{79.1° \mp \alpha°}{2}$

This is dependent upon one angle being correct, which must be in degrees, for $\arctan\left(3\sqrt{3}\right)$

$\tan\left(2\theta + 60°\right) = 3\sqrt{3} \Rightarrow \theta = 9.6°$ would imply B1 M1 dM1

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range

Otherwise: Via the use of $\cos\left(2\theta + 30°\right) = \cos 2\theta\cos 30° - \sin 2\theta\sin 30°$.

$2\sin 2\theta = \cos\left(2\theta + 30°\right) \Rightarrow \tan 2\theta = \frac{\sqrt{3}}{5} \Rightarrow \theta = 9.6°, 99.6°$

The order of the marks needs to match up to the main scheme so 0110 is possible.

B1: For achieving $\tan 2\theta = \frac{\sqrt{3}}{5}$ o.e so allow $\tan 2\theta =$ awrt 0.346 or $\tan 2\theta = \frac{\cos 30°}{2 + \sin 30°}$

Or via double angle identities $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ o.e.

M1: Attempts to use the compound angle identities to reach a form $\tan 2\theta = k$ where k is a constant not $3\sqrt{3}$ (or expression in trig terms such as cos 30 etc as seen above)

Or via double angle identities reaches a 3TQ in $\tan\theta$

dM1: Correct order of operations from $\tan 2\theta = k$ leading to $\theta = ...$

Correctly solves their $\sqrt{3}\tan^2\theta + 10\tan\theta - \sqrt{3} = 0$ leading to $\theta = ...$

A1: $\theta =$ awrt $9.6°, 99.6°$ with no other values given in the range.

Note that $\tan\left(2\theta + 60°\right) = 3\sqrt{3} \Rightarrow \theta = 9.6°, 99.6°$ is acceptable for full marks

Question	Scheme	Marks	AOs
3(a)	Uses the common ratio $\frac{5+2\sin\theta}{12\cos\theta}=\frac{6\tan\theta}{5+2\sin\theta}$ o.e.	M1	3.1a
	Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\times12\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(a) Alt	**(a) Alternative example:**		
	Uses the common ratio $12r\cos\theta=5+2\sin\theta,\ 12r^2\cos\theta=6\tan\theta$ $\Rightarrow 12\cos\theta\left(\frac{5+2\sin\theta}{12\cos\theta}\right)^2=6\tan\theta$	M1	3.1a
	Multiplies up and uses $\tan\theta\times\cos\theta=\sin\theta$ $(5+2\sin\theta)^2=6\tan\theta\times12\cos\theta=72\sin\theta$	dM1	1.1b
	Proceeds to given answer $25+20\sin\theta+4\sin^2\theta=72\sin\theta$ $\Rightarrow 4\sin^2\theta-52\sin\theta+25=0$ *	A1*	2.1
		(3)	
(b)	$4\sin^2\theta-52\sin\theta+25=0\Rightarrow\sin\theta=\frac{1}{2}\left(,\frac{25}{2}\right)$	M1	1.1b
	$\theta=\frac{5\pi}{6}$	A1	1.2
		(2)	
(c)	Attempts a value for either *a* or *r* e.g. $a=12\cos\theta=\ 12\times-\frac{\sqrt{3}}{2}$ **or** $r=\frac{5+2\sin\theta}{12\cos\theta}=\frac{5+2\times\frac{1}{2}}{12\times-\frac{\sqrt{3}}{2}}$	M1	3.1a
	$"a"=-6\sqrt{3}$ **and** $"r"=-\frac{1}{\sqrt{3}}$ o.e.	A1	1.1b
	Uses $S_\infty=\frac{a}{1-r}=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$	dM1	2.1
	Rationalises denominator $S_\infty=\frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}=\frac{-18}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$	ddM1	1.1b
	$(S_\infty=)9(1-\sqrt{3})$	A1	2.1
		(5)	
		(10 marks)	

Notes:

(a)

M1: For the key step in using the ratio of $\frac{a_2}{a_1}=\frac{a_3}{a_2}$

dM1: Cross multiplies and uses $\tan\theta\times\cos\theta=\sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Alternative:

M1: Expresses the 2nd and 3rd terms in terms of the first term and the common ratio and eliminates "r"

dM1: Multiplies up and uses $\tan\theta \times \cos\theta = \sin\theta$

A1*: Proceeds to the given answer including the " = 0" with no errors and sufficient working shown.

Other approaches may be seen in (a) and can be marked in a similar way e.g. M1 for correctly obtaining an equation in θ using the GP, M1 for applying $\tan\theta \times \cos\theta = \sin\theta$ or equivalent and eliminating fractions, A1 as above

Example: $u_2 = \frac{u_1 \times u_3}{u_2} \Rightarrow 5 + 2\sin\theta = \frac{12\cos\theta \times 6\tan\theta}{5 + 2\sin\theta}$	**M1**
$\Rightarrow (5 + 2\sin\theta)^2 = 72\sin\theta$	**dM1**
$25 + 20\sin\theta + 4\sin^2\theta = 72\sin\theta$	
$\Rightarrow 4\sin^2\theta - 52\sin\theta + 25 = 0$ *	**A1**

(b)

M1: Attempts to solve $4\sin^2\theta - 52\sin\theta + 25 = 0$. Must be clear they have found $\sin\theta$ and not e.g. just x from $4x^2 - 52x + 25 = 0$. Working does not need to be seen but see general guidance for solving a 3TQ if necessary. Note that the $\frac{25}{2}$ does not need to be seen.

A1: $\theta = \frac{5\pi}{6}$ and no other values unless they are rejected or the $\frac{5\pi}{6}$ clearly selected here and not in (c)

A minimum requirement in (b) is e.g. $\sin\theta = \frac{1}{2}$, $\theta = \frac{5\pi}{6}$

Do **not** allow 150° for $\frac{5\pi}{6}$

PTO for the notes to part (c)

(c) Allow full marks in (c) if e.g. $\theta = \frac{\pi}{6}$ is their answer to (b) but $\theta = \frac{5\pi}{6}$ is used here.

or if e.g. $\theta = \frac{5\pi}{6}$ is their answer to (b) but $\theta = \frac{\pi}{6}$ is used here allow the M marks only.

M1: For attempting a value (exact or decimal) for either a or r using **their** θ

E.g. $a = 12\cos\theta = \left(12 \times -\frac{\sqrt{3}}{2}\right)$ or $r = \frac{5+2\sin\theta}{12\cos\theta} = \left(\frac{5+2\times\frac{1}{2}}{12\times -\frac{\sqrt{3}}{2}}\right)$ oe e.g. $r = \frac{6\tan\theta}{5+2\sin\theta} = \left(\frac{6\times -\frac{1}{\sqrt{3}}}{5+2\times\frac{1}{2}}\right)$

A1: Finds both $a = -6\sqrt{3}$ and $r = -\frac{1}{\sqrt{3}}$ which can be left unsimplified but $\sin\theta = \frac{1}{2}, \cos\theta = -\frac{\sqrt{3}}{2}$

and $\tan\theta = -\frac{\sqrt{3}}{3}$ (if required) must have been used.

dM1: Uses both **values** of "a" and "r" with the equation $S_\infty = \frac{a}{1-r} = \frac{-6\sqrt{3}}{1+\frac{1}{\sqrt{3}}}$ to create an expression

involving surds where a and r have come from appropriate work and $|r| < 1$

Depends on the first method mark.

ddM1: Rationalises denominator. The denominator must be of the form $p \pm q\sqrt{3}$ oe e.g. $p + \frac{q}{\sqrt{3}}$

Depends on both previous method marks.

Note that stating e.g. $\frac{k}{p+q\sqrt{3}} \times \frac{p-q\sqrt{3}}{p-q\sqrt{3}}$ or $\frac{k}{p+\frac{q}{\sqrt{3}}} \times \frac{p-\frac{q}{\sqrt{3}}}{p-\frac{q}{\sqrt{3}}}$ is sufficient.

A1: Obtains $(S_\infty =) 9(1-\sqrt{3})$

Note that full marks are available in (c) for the use of $\theta = 150°$

Note also that marks may be implied in (c) by e.g.

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\theta}{1-\frac{5+2\sin\theta}{12\cos\theta}} = \frac{144\cos^2\theta}{12\cos\theta - 5 - 2\sin\theta} = \frac{144\cos^2\frac{5\pi}{6}}{12\cos\frac{5\pi}{6} - 5 - 2\sin\frac{5\pi}{6}}$$

$$= \frac{108}{-6-6\sqrt{3}} = \frac{108}{-6-6\sqrt{3}} \times \frac{-6+6\sqrt{3}}{-6+6\sqrt{3}} = \frac{-648+648\sqrt{3}}{-72} = 9(1-\sqrt{3})$$

Scores M1A1 implied dM1 ddM1 A1

See next page for some other cases in (c) and how to mark them:

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} \quad \text{or e.g.} \quad S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}}$$

And nothing else

scores M1A0dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{5\pi}{6}}{1-\frac{5+2\sin\frac{5\pi}{6}}{12\cos\frac{5\pi}{6}}} = 9\left(1-\sqrt{3}\right)$$

Scores M1A1dM1ddM0A0

$$S_\infty = \frac{a}{1-r} = \frac{12\cos\frac{\pi}{6}}{1-\frac{5+2\sin\frac{\pi}{6}}{12\cos\frac{\pi}{6}}} = 9\left(1+\sqrt{3}\right)$$

Scores M1A0dM1ddM0A0

$S_\infty = 9\left(1-\sqrt{3}\right)$ with no working scores no marks

Question	Scheme	Marks	AOs
4(a)	$R=\sqrt{2^2+8^2}=\sqrt{68}=2\sqrt{17}$	B1	1.1b
	$2\cos\theta+8\sin\theta=R\cos\theta\cos\alpha+R\sin\theta\sin\alpha$ $2=R\cos\alpha \quad 8=R\sin\alpha$ $\tan\alpha=\frac{8}{2}\Rightarrow\alpha=...$	M1	1.1b
	$\alpha=$ awrt 1.326	A1	2.2a
		(3)	
(b)(i)	$4.5\times"2\sqrt{17}"$	M1	1.1b
	$9\sqrt{17}$	A1	2.2a
(ii)	awrt 1.33	B1ft	2.2a
		(3)	
			(6 marks)

Notes

(a)

B1: $R=2\sqrt{17}$ or $\sqrt{68}$.

$\pm2\sqrt{17}$ or $\pm\sqrt{68}$ score B0

(Condone if this comes from e.g., $8=R\cos\alpha \quad 2=R\sin\alpha$)

Decimal answers score B0 unless the exact value is seen then apply isw.

M1: Proceeds to a value for α from $\tan\alpha=\pm\frac{8}{2}$, $\cos\alpha=\pm\frac{2}{"\sqrt{68}"}$, $\sin\alpha=\pm\frac{8}{"\sqrt{68}"}$

May be implied by awrt 1.33 radians or 76 degrees

A1: awrt 1.326 for α. Apply isw if this is then subsequently rounded to e.g. 1.33

(b)(i)

M1: For a value of $\pm4.5\times$ their R or allow $\pm 4.5R$ (with the letter R)

But not embedded in an expression e.g. $9\sqrt{17}\cos(\theta-\alpha)$ unless extracted later.

Note that the sum may be found as $9\cos x+36\sin x$ with the maximum then found using calculus

e.g. $S=9\cos x+36\sin x\Rightarrow\frac{dS}{dx}=-9\sin x+36\cos x=0\Rightarrow\tan x=4\Rightarrow\sin x=\frac{4}{\sqrt{17}},\ \cos x=\frac{1}{\sqrt{17}}$

$\Rightarrow9\cos x+36\sin x=9\sqrt{17}$. This will score M1 once they reach $\pm4.5\times$ their R

May be implied by $9\sqrt{17}$ or awrt 37.1 (which may come from a graphical method)

May also see e.g. $\text{Max}(9\cos x+36\sin x)=\sqrt{9^2+36^2}=...$

A1: $9\sqrt{17}$ or exact equivalent e.g. $\sqrt{1377}$, $4.5\sqrt{68}$, $4.5(2\sqrt{17})$ and apply isw once a correct answer is seen

(ii)

B1ft: awrt 1.33 (or follow through on their α even if in degrees (76), no matter how accurate)

Question	Scheme	Marks	AOs
5(a)	e.g. $2\frac{\sin\theta}{\cos\theta}(8\cos\theta+23\underline{(1-\cos^2\theta)})=8\times\underline{2\sin\theta\cos\theta\sec^2\theta}$	B1	1.2
	$2\tan\theta(8\cos\theta+23\sin^2\theta)=8\sin 2\theta\sec^2\theta$ $\Rightarrow 2\sin\theta\cos\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin 2\theta$ $\sin 2\theta(8\cos\theta+23(1-\cos^2\theta))=8\sin 2\theta$ $\sin 2\theta(23\cos^2\theta-8\cos\theta-15)=0$	M1A1	2.1 2.2a
		(3)	
(b)	$\sin 2x(23\cos^2 x-8\cos x-15)=0$		
	$\sin 2x=0\Rightarrow x=360°$ **or** $540°$	B1	2.2a
	$23\cos^2 x-8\cos x-15\Rightarrow\cos x=-\frac{15}{23}$	M1	1.1b
	$\cos x=-\frac{15}{23}\Rightarrow x=...$	dM1	1.1b
	$x=360°, 540°$ and awrt $491°$ only	A1	2.3
		(4)	
			(7 marks)

Notes

(a) Allow use of e.g. *x* but the final mark requires the equation to be in terms of *θ*

B1(M1 on EPEN): For recalling and using at least one correct trigonometric identity in the given equation.

e.g. one of: $\sin^2\theta+\cos^2\theta=1,\ 1+\tan^2\theta=\sec^2\theta,\ \tan\theta=\frac{\sin\theta}{\cos\theta},\ \sin 2\theta=2\sin\theta\cos\theta$

This may be seen explicitly or may be implied by their working by e.g. $\tan\theta\cos\theta=\sin\theta$ or they might multiply both sides by $\cos^2\theta$ leaving $8\sin 2\theta$ on the rhs implying $1+\tan^2\theta=\sec^2\theta$

M1: For manipulating the equation using trigonometric identities (condoning sign slips only in the identities and arithmetic slips) to obtain an expression of the form:

$A\sin 2\theta\cos^2\theta+B\sin 2\theta\cos\theta+C\sin 2\theta\ (=0)$ or $\sin 2\theta\left(A\cos^2\theta+B\cos\theta+C\right)\ (=0)$ with $A, B, C\neq 0$

A1: $\sin 2\theta(23\cos^2\theta-8\cos\theta-15)=0$ oe e.g. $\sin 2\theta(-23\cos^2\theta+8\cos\theta+15)=0$ cao

Note that this is not a given answer so condone notational slips e.g. $\cos\theta^2$ for $\cos^2\theta$ provided the intention is clear but the final equation must have no notational errors.

Note that the "= 0" is not required for the M1 but is required for the A1

Note: some candidates arrive at the correct final answer fortuitously following errors in their work.

(b) Allow all marks in (b) to score if the correct equation is obtained fortuitously in part (a)

Also allow use of *θ* instead of *x* throughout in part (b). Correct answers, no working scores max 1000

B1: For one of $x=360(°)$ or $x=540(°)$ Condone $x=2\pi$ or $x=3\pi$ for this mark.

The degrees symbol is not required. This may come from $\cos x=1$

M1: Attempts to solve their 3TQ from part (a) or a "made up" 3TQ (which may only be seen in (b)) leading to a value for $\cos x$. The general guidance for solving a 3 term quadratic equation can be applied. Allow solution(s) from a calculator which may be implied by at least one correct value for their 3TQ. Must be a value for $\cos x$ and not e.g. x.

dM1: Attempts to find one of their angles in the range $360<x<540$ (but not 450) for their $\cos x=k$ where $|k|<1$ May be implied by their value(s) but must be in degrees.

Requires them to state a value for $\cos x$. Must be checked (you can check cos(their x) = their k (1sf))

A1: $x=360°, 540°$ and awrt $491°$ only with no other values in range (including 450).

The degrees symbol is not required. awrt 491 must come from $\cos x=-\frac{15}{23}$