**1.** **In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Given that $1 + \cos 2\theta + \sin 2\theta \neq 0$ prove that

$$\frac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta} \equiv \tan \theta$$

**(4)**

(b) Hence solve, for $0 < x < 180°$

$$\frac{1 - \cos 4x + \sin 4x}{1 + \cos 4x + \sin 4x} = 3 \sin 2x$$

giving your answers to one decimal place where appropriate.

**(4)**

**2.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

**(4)**

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

**(4)**

**3.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

Given that the first three terms of a geometric series are

$$12\cos\theta \qquad 5 + 2\sin\theta \qquad \text{and} \qquad 6\tan\theta$$

(a) show that

$$4\sin^2\theta - 52\sin\theta + 25 = 0$$

**(3)**

Given that $\theta$ is an obtuse angle measured in radians,

(b) solve the equation in part (a) to find the exact value of $\theta$

**(2)**

(c) show that the sum to infinity of the series can be expressed in the form

$$k\left(1 - \sqrt{3}\right)$$

where $k$ is a constant to be found.

**(5)**

**4.** (a) Express $2\cos\theta + 8\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R$ and $\alpha$ are constants,

$R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of $R$ and give the value of $\alpha$ in radians to 3 decimal places.

**(3)**

The first three terms of an arithmetic sequence are

$$\cos x \qquad \cos x + \sin x \qquad \cos x + 2\sin x \qquad x \neq n\pi$$

Given that $S_9$ represents the sum of the first 9 terms of this sequence as $x$ varies,

(b) (i) find the exact maximum value of $S_9$

(ii) deduce the smallest positive value of $x$ at which this maximum value of $S_9$ occurs.

**(3)**

**5.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$2\tan\theta(8\cos\theta + 23\sin^2\theta) = 8\sin 2\theta(1 + \tan^2\theta)$$

may be written as

$$\sin 2\theta(A\cos^2\theta + B\cos\theta + C) = 0$$

where $A$, $B$ and $C$ are constants to be found.

**(3)**

(b) Hence, solve for $360° \leqslant x \leqslant 540°$

$$2\tan x(8\cos x + 23\sin^2 x) = 8\sin 2x(1 + \tan^2 x) \qquad x \in \mathbb{R} \qquad x \neq 450°$$

**(4)**