1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Given that

$$2\sin(x - 60°) = \cos(x - 30°)$$

show that

$$\tan x = 3\sqrt{3}$$

(4)

(b) Hence or otherwise solve, for $0 \leqslant \theta < 180°$

$$2\sin 2\theta = \cos(2\theta + 30°)$$

giving your answers to one decimal place.

(4)

from formula book:

$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$

$\cos(A \pm B) = \sin A \sin B \mp \cos A \cos B$

(a) $\sin(x-60) = \sin x \cos 60 - \sin 60 \cos x$

$\cos(x-30) = \sin x \sin 30 + \cos x \cos 30$ (1)

$2\sin x \cos 60 - 2\sin 60 \cos x = \sin x \sin 30 + \cos x \cos 30$ (1)

$\sin 60 = \frac{\sqrt{3}}{2}$ $\quad \sin 30 = \frac{1}{2}$ $\quad \cos 60 = \frac{1}{2}$ $\quad \cos 30 = \frac{\sqrt{3}}{2}$

$\left(2\sin x \times \frac{1}{2}\right) - \left(2 \times \frac{\sqrt{3}}{2} \times \cos x\right) = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$ (1)

collect sin and cos terms

$\sin x - \sqrt{3}\cos x = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x$

$\frac{1}{2}\sin x = \left(\frac{\sqrt{3}}{2} + \sqrt{3}\right)\cos x$

$\times 2$

$\sin x = \sqrt{3} + 2\sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = \tan x$

$\sin x = 3\sqrt{3}\cos x$

$\frac{\cos x}{\cos x} = 1$

$\tan x = 3\sqrt{3}$ (1)

(b) Using part (a):

$x - 60 = 2\theta$ $\qquad$ $x - 30 = 2\theta + 30$

$x = 2\theta + 60$ (1) $\qquad$ $x = 2\theta + 60$

So we can use $\tan(x) = 3\sqrt{3}$ with $x = 2\theta + 60$

$\tan(2\theta + 60) = 3\sqrt{3}$

$2\theta + 60 = \tan^{-1}(3\sqrt{3})$

$2\theta + 60 = 79.1$ (1)

$2\theta = 19.1$

$\theta = 9.6$ (1) $\leftarrow$ tan repeats every $90°$

$\theta = 9.6°, 99.6°$ (1)

$9.6 + 90 = 99.6$

the next value (189.6) will be outside the given range of $0 \leq \theta \leq 180$