

Question	Scheme	Marks	AOs	
1(a)	$\frac{1}{\cos \theta} + \tan \theta = \frac{1 + \sin \theta}{\cos \theta} \text{ or } \frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta}$	M1	1.1b	
	$= \frac{1 + \sin \theta}{\cos \theta} \times \frac{1 - \sin \theta}{1 - \sin \theta} = \frac{1 - \sin^2 \theta}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta}{\cos \theta (1 - \sin \theta)}$ or $\frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta} = \frac{(1 + \sin \theta) \cos \theta}{1 - \sin^2 \theta} = \frac{(1 + \sin \theta) \cos \theta}{(1 + \sin \theta)(1 - \sin \theta)}$	dM1	2.1	
	$= \frac{\cos \theta}{1 - \sin \theta} *$	A1*	1.1b	
		(3)		
(b)	$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$ $\Rightarrow 1 + \sin 2x = 3 \cos^2 2x = 3(1 - \sin^2 2x)$	$\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x$ $\Rightarrow \cos 2x = 3 \cos 2x (1 - \sin 2x)$	M1	2.1
	$\Rightarrow 3 \sin^2 2x + \sin 2x - 2 = 0$	$\Rightarrow \cos 2x (2 - 3 \sin 2x) = 0$	A1	1.1b
	$\sin 2x = \frac{2}{3}, (-1) \Rightarrow 2x = \dots \Rightarrow x = \dots$		M1	1.1b
	$x = 20.9^\circ, 69.1^\circ$		A1	1.1b
			A1	1.1b
			(5)	
(8 marks)				

Notes

(a) If starting with the LHS: Condone if another variable for θ is used except for the final mark

M1: Combines terms with a common denominator. The numerator must be correct for their common denominator.

dM1: Either:

- $\frac{1 + \sin \theta}{\cos \theta}$: Multiplies numerator and denominator by $1 - \sin \theta$, uses the difference of two squares and applies $\cos^2 \theta = 1 - \sin^2 \theta$
- $\frac{(1 + \sin \theta) \cos \theta}{\cos^2 \theta}$: Uses $\cos^2 \theta = 1 - \sin^2 \theta$ on the denominator, applies the difference of two squares

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. Withhold this mark for writing eg \sin instead of $\sin \theta$ anywhere in the solution and for eg $\sin \theta^2$ instead of $\sin^2 \theta$

Alt(a) If starting with the RHS: Condone if another variable is used for θ except for the final mark

M1: Multiplies by $\frac{1 + \sin \theta}{1 + \sin \theta}$ leading to $\frac{\cos \theta(1 + \sin \theta)}{1 - \sin^2 \theta}$ or

Multiplies by $\frac{\cos \theta}{\cos \theta}$ leading to $\frac{\cos^2 \theta}{\cos \theta(1 - \sin \theta)}$

dM1: Applies $\cos^2 \theta = 1 - \sin^2 \theta$ and cancels the $\cos \theta$ factor from the numerator and denominator leading to $\frac{1 + \sin \theta}{\cos \theta}$ or

Applies $\cos^2 \theta = 1 - \sin^2 \theta$ and uses the difference of two squares leading to $\frac{(1 + \sin \theta)(1 - \sin \theta)}{\cos \theta(1 - \sin \theta)}$

It is dependent on the previous method mark.

A1*: Fully correct proof with correct notation and no errors in the main body of their work. If they work from both the LHS and the RHS and meet in the middle with both sides the same then they need to conclude at the end by stating the original equation.

(b) ***Be aware that this can be done entirely on their calculator which is not acceptable***

M1: Either multiplies through by $\cos 2x$ and applies $\cos^2 2x = 1 - \sin^2 2x$ to obtain an equation in $\sin 2x$ only or alternatively sets $\frac{\cos 2x}{1 - \sin 2x} = 3 \cos 2x$ and multiplies by $1 - \sin 2x$

A1: Correct equation or equivalent. The $= 0$ may be implied by their later work (Condone notational slips in their working)

M1: Solves for $\sin 2x$, uses arcsin to obtain at least one value for $2x$ and divides by 2 to obtain at least one value for x . The roots of the quadratic can be found using a calculator. They cannot just write down values for x from their quadratic in $\sin 2x$

A1: For 1 of the required angles. Accept awrt 21 or awrt 69. Also accept awrt 0.36 rad or awrt 1.21 rad

A1: For both angles (awrt 20.9 and awrt 69.1) and no others inside the range. If they find $x = 45$ it must be rejected. (Condone notational slips in their working)

Question	Scheme	Marks	AOs
2(a)	States or uses $\tan x = \frac{\sin x}{\cos x}$	B1	1.2
	$4 \sin x = 5 \cos^2 x \Rightarrow 4 \sin x = 5(1 - \sin^2 x)$	M1	1.1b
	$5 \sin^2 x + 4 \sin x - 5 = 0^*$	A1*	2.1
		(3)	
(b)	Attempts to solve $5 \sin^2 x + 4 \sin x - 5 = 0 \Rightarrow \sin x = \dots$	M1	1.1b
	$\sin x = \frac{-2 \pm \sqrt{29}}{5}$ ($\sin x = \text{awrt } 0.677$)	A1	1.1b
	Takes \sin^{-1} leading to at least one answer in the range	dM1	1.1b
	$x = \text{awrt } 42.6\{\circ\}$ and $x = \text{awrt } 137.4\{\circ\}$ only	A1	1.1b
		(4)	
(c)	$15 \times "2" = 30$ following through on their "2"	B1ft	2.2a
	Explains either "mathematically" by stating $3 \times 5 \times$ their number in range 0 to 360° or 'in words" e.g., stating $3 \times "2"$ values every 360° and 5 lots of 360°	B1ft	2.4
		(2)	

(9 marks)**Notes:****(a) Allow use of e.g. θ but the final mark requires the equation to be in terms of x** **B1:** States or uses $\tan x = \frac{\sin x}{\cos x}$ e.g., $4 \tan x = 5 \cos x \Rightarrow 4 \frac{\sin x}{\cos x} = 5 \cos x$ Allow e.g. $\tan x = \frac{\sin \theta}{\cos \theta}$ **M1:** Multiplies by $\cos x$ and uses $\cos^2 x = 1 - \sin^2 x$ to set up a quadratic equation in just $\sin x$
Condone mixed arguments here.**A1*:** Proceeds to $5 \sin^2 x + 4 \sin x - 5 = 0$ with correct notation and algebra, showing all key steps.
The $= 0$ must be present in the final answer line.Condone a single slip in notation, e.g., $\sin x^2$ or $\sin \theta$ seen once.**(b)****M1:** Attempts to solve $5 \sin^2 x + 4 \sin x - 5 = 0 \Rightarrow \sin x = \dots$ using the usual rules.
 $\sin x =$ may be implied later.Allow solution(s) from a calculator but one must be correct (0.6 or 0.7 or -1.4 or -1.5)**A1:** Achieves $\sin x = \frac{-4 \pm \sqrt{116}}{10}$ ($\sin x = \text{awrt } 0.677$) $\sin x =$ may be implied later.**dM1:** Finds one value of x in the range 0 to 360° from their $\sin x =$ May be scored for working in radians. If using $\sin x = 0.677$ they should have awrt 0.744 or awrt 2.40

If they have made a slip in solving the quadratic, e.g., by the formula, then their values will need checking both in degrees and radians to see if this mark can be implied.

A1: $x = \text{awrt } 42.6\{\circ\}$ and $x = \text{awrt } 137.4\{\circ\}$ only. Ignore any values outside of 0 to 360° isw if they round their values to e.g., 3sf after stating acceptable answers.
There must be some evidence that the quadratic has been solved.

(c)

B1ft: Follow through on 15 multiplied by the number of solutions in (b) in the range 0 to 360°

If working in radians in (b), they must state 30 (solutions).

B1ft: Explains either mathematically **or** in words. See scheme.

Note that you might see arguments expanding the range from 1800 to 5400 to account for the stretch parallel to the x axis. $\frac{5400}{360} = 15$ and $15 \times 2 = 30$ which is also acceptable.

Note: If candidates list 30 values and conclude that there are 30 solutions, score B1ftB1ft
There is no need to check their 30 values are correct, but there must be 30.