Name:

Exam Style Questions

Simultaneous Equations Linear and Non-linear

Corbettmaths

Equipment needed: Pen, Calculator

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Video Tutorial

www.corbettmaths.com/contents

Video 298

Answers and Video Solutions

1. Solve the equations

$$x^2 + y^2 = 20$$

$$x + y = 6$$

$x = 6 - y$

$(6-y)^2 + y^2 = 20$

$(6-y)(6-y) + y^2 = 20$

$36 - 12y + y^2 + y^2 = 20$

$2y^2 - 12y + 16 = 0$

$y^2 - 6y + 8 = 0$

$(y-2)(y-4) = 0$

$y = 2 \quad y = 4$

when $y = 2$, $x = 4$

when $y = 4$, $x = 2$

$x = 2$ and $y = 4$

or $x = 4$ and $y = 2$

(4)

2. Solve the equations

$$xy = 24$$

$$x = y - 2$$

$(y-2)y = 24$

$y^2 - 2y = 24$

$y^2 - 2y - 24 = 0$

$(y-6)(y+4) = 0$

$y = 6$ or $y = -4$

when $y = 6$

$x = 6 - 2$

$x = 4$

when $y = -4$

$x = -4 - 2$

$= -6$

$x = 4$ and $y = 6$

or $x = -6$ and $y = -4$

(4)

3. Solve the simultaneous equations

$$2x - y = 7$$

$$xy = 15$$

$2x = y + 7$

$y = 2x - 7$

$x(2x - 7) = 15$

$2x^2 - 7x - 15 = 0$

$(2x + 3)(x - 5) = 0$

$2x + 3 = 0$ or $x = 5$

$2x = -3$

$x = -1.5$

when $x = 5$

$2x - y = 7$

$10 - y = 7$

$y = 3$

when $x = -1.5$

$2x - y = 7$

$-3 - y = 7$

$y = -10$

$x = 5$ and $y = 3$

or

$x = -1.5$ and $y = -10$

(4)

4. Solve the equations

$$x^2 + y^2 = 17$$

$$x + 4y = 0$$

$x = -4y$

$(-4y)^2 + y^2 = 17$

$16y^2 + y^2 = 17$

$17y^2 = 17$

$y^2 = 1$

$y = 1$ or $y = -1$

when $y = 1$

$x + 4 = 0$

$x = -4$

when $y = -1$

$x - 4 = 0$

$x = 4$

$x = -4$ and $y = 1$

or

$x = 4$ and $y = -1$

(4)

5. Solve the equations

$$x + 2y = 3$$

$$x^2 + 3xy = 10$$

$x = 3 - 2y$

$(3-2y)^2 + 3(3-2y)y = 10$

$(3-2y)(3-2y) + 3y(3-2y) = 10$

$9 - 12y + 4y^2 + 9y - 6y^2 = 10$

$-2y^2 - 3y + 9 = 10$

$-2y^2 - 3y - 1 = 0$

$2y^2 + 3y + 1 = 0$

$(2y+1)(y+1) = 0$

$y = -0.5$ or $y = -1$

when $y = -0.5$

$x = 4$

when $y = -1$

$x = 5$

$x = 4$ and $y = -0.5$

or $x = 5$ and $y = -1$

(4)

6. Solve the equations

$$2x + y = 11$$

$$2x^2 - y^2 = 23$$

$y = 11 - 2x$

$2x^2 - (11-2x)^2 = 23$

$2x^2 - (121 - 44x + 4x^2) = 23$

$-2x^2 + 44x - 121 = 23$

$-2x^2 + 44x - 144 = 0$

$2x^2 - 44x + 144 = 0$

$x^2 - 22x + 72 = 0$

$(x-4)(x-18) = 0$

$x = 4$ or $x = 18$

$x = 4$

$8 + y = 11$

$y = 3$

$x = 18$

$36 + y = 11$

$y = -25$

$x = 4$ and $y = 3$

or

$x = 18$ and $y = -25$

(4)

7. Solve the equations

$$x^2 + y^2 = 25$$

$$x + y = 7$$

$x = 7 - y$

$(7-y)^2 + y^2 = 25$

$49 - 14y + y^2 + y^2 = 25$

$2y^2 - 14y + 24 = 0$

$y^2 - 7y + 12 = 0$

$(y-3)(y-4) = 0$

$y = 3$ or $y = 4$

when $y = 3$
$x = 4$

or when $y = 4$
$x = 3$

$x = 4$ and $y = 3$
or
$x = 3$ and $y = 4$

(5)

8. Find the coordinates of the points where the line $y = 2x - 2$ and the curve $y = x^2 - 5$ intersect.

$x^2 - 5 = 2x - 2$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3$ or $x = -1$

when $x = 3$ $y = 6 - 2$
$y = 4$

when $x = -1$ $y = -4$

(3, 4) and (-1, -4)

(5)

9. Solve the equations

$$x^2 - y^2 = 7$$

$$2y = 2 + x$$

$x = 2y - 2$

$(2y-2)^2 - y^2 = 7$

$(2y-2)(2y-2) - y^2 = 7$

$4y^2 - 8y + 4 - y^2 = 7$

$3y^2 - 8y - 3 = 0$

$(3y+1)(y-3) = 0$

$y = -\frac{1}{3}$ or $y = 3$

$y = 3$

$6 = 2 + x$

$x = 4$

$y = -\frac{1}{3}$

$-\frac{2}{3} = 2 + x$

$x = -2\frac{2}{3}$

$x = 4$, $y = 3$

or $x = -2\frac{2}{3}$, $y = -\frac{1}{3}$

(5)

10. How many points of intersection does the circle $x^2 + y^2 = 18$ have with the line $x + y = 6$?

$x^2 + y^2 = 18$

$x = 6 - y$

$(6-y)^2 + y^2 = 18$

$36 - 12y + y^2 + y^2 = 18$

$18 - 12y + 2y^2 = 0$

$9 - 6y + y^2 = 0$

$y^2 - 6y + 9 = 0$

$(y-3)(y-3) = 0$

$y = 3$

$y = 3$

$x = 3$

only $(3, 3)$

(5)

11. Solve the equations

$$x^2 + y^2 = 45$$

$$5x - 3y = 21$$

Give your answers to an appropriate degree of accuracy.

$5x = 3y + 21$

$3y = 5x - 21$

$y = \frac{5}{3}x - 7$

$y = \frac{5}{3}x - 7$

when $x = 6$

$y = 3$

when $x = 0.176...$

$y = -6.7...$

$x^2 + \left(\frac{5}{3}x - 7\right)^2 = 45$

$x^2 + \left(\frac{5}{3}x - 7\right)\left(\frac{5}{3}x - 7\right) = 45$

$x^2 + \frac{25}{9}x^2 - \frac{35}{3}x - \frac{35}{3}x + 49 = 45$

$\frac{34}{9}x^2 - \frac{70}{3}x + 4 = 0 \quad \times 9$

$34x^2 - 210x + 36 = 0 \quad \div 2$

$17x^2 - 105x + 18 = 0$

$a = 17$

$b = -105$

$c = 18$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{105 \pm \sqrt{11025 - 1224}}{34}$

$x = \frac{105 \pm 99}{34}$

$x = 6$ or $x = 0.176...$

$x = 0.2$ & $y = -6.7$

or $x = 6$ & $y = 3$

.....

(5)

12. Find the coordinates of the points where the line $y = 3x - 3$ and the curve $y = 2x^2 - 8x + 2$ intersect.

$2x^2 - 8x + 2 = 3x - 3$

$2x^2 - 11x + 5 = 0$

$(2x - 1)(x - 5) = 0$

$x = \frac{1}{2}$ or $x = 5$

$y = 3x - 3$

When $x = \frac{1}{2}$

$y = -1.5$

When $x = 5$

$y = 12$

$(\frac{1}{2}, -\frac{3}{2})$ and $(5, 12)$

(5)

13. Solve the equations

$$y = 3x^2 + 8x - 9$$

$$2x + y = 15$$

Give your answers to 1 decimal place.

$2x + 3x^2 + 8x - 9 = 15$

$3x^2 + 10x - 24 = 0$

$a = 3 \quad b = 10 \quad c = -24$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{100 - (-288)}}{6}$$

$$x = \frac{-10 \pm \sqrt{388}}{6}$$

$x = \frac{-10 + \sqrt{388}}{6}$ or $x = \frac{-10 - \sqrt{388}}{6}$

$x = 1.616\ldots$ $x = -4.949\ldots$

$x = 1.6$ $x = -4.9$

$x = 1.6$ & $y = 11.8$

or

$x = -4.9$ & $y = 24.9$

(5)

$y = 15 - 2x$

when $x = 1.616\ldots$ $y = 11.767\ldots$

when $x = -4.94\ldots$ $y = 24.899\ldots$