

**MARK SCHEME for the May/June 2012 question paper  
for the guidance of teachers**

**9231 FURTHER MATHEMATICS**

**9231/11**

Paper 1, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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## Mark Scheme Notes

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\surd$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu No	Commentary	Solution	Marks	Part Mark	Total
1	States $\sum \alpha$ and $\sum \alpha\beta$ Uses formula for correctly.  Uses formula for $\sum \alpha^3$ to obtain result.	$\sum \alpha = 7 \quad \sum \alpha\beta = 2$ $\sum \alpha^2 = 7^2 - 2 \times 2 = 45$  $\sum \alpha^3 = 7 \sum \alpha^2 - 2 \sum \alpha + 9$ $= 315 - 14 + 9 = 310$	B1 B1  M1 A1A1	2  3	[5]
2	(States proposition.)  Proves base case.  States inductive hypothesis. Proves inductive step.  States conclusion.	( $P_n: 4^n > 2^n + 3^n$ )  Let $n = 2$ , $16 > 4 + 9 \Rightarrow P_2$ is true.  Assume $P_k$ is true $\Rightarrow 4^k > 2^k + 3^k$  $4^{k+1} = 4 \cdot 4^k > 4(2^k + 3^k) = 4 \cdot 2^k + 4 \cdot 3^k$ $> 2 \cdot 2^k + 3 \cdot 3^k = 2^{k+1} + 3^{k+1}$ $\therefore P_k \Rightarrow P_{k+1}$  Hence result true, by PMI, for all integers $n \geq 2$ .	B1 B1 M1 A1  A1 (CWO)	5	[5]
3	Proves initial result.  Sets up method of differences.  Shows cancellation to get result.  States sum to infinity.	$f(r-1) - f(r) = \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$ $= \frac{r+2-r}{r(r+1)(r+2)} = \frac{2}{r(r+1)(r+2)} \quad (\text{AG})$ $\sum_1^n \frac{1}{r(r+1)(r+2)} = \frac{1}{2} \left\{ \frac{1}{1 \times 2} - \frac{1}{2 \times 3} \right\} \dots \dots$ $+ \frac{1}{2} \left\{ \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \right\}$ $= \frac{1}{4} - \frac{1}{2} \left\{ \frac{1}{(n+1)(n+2)} \right\} \quad (\text{OE})$ $\therefore \sum_1^\infty \frac{1}{r(r+1)(r+2)} = \frac{1}{4}$	M1 A1  M1A1  A1 A1√	2  3 1	[6]
	<b>'Non hence' method for last two parts</b>  i.e. penalty of 1 mark.	$\frac{1}{r(r+1)(r-2)} = \frac{1}{2r} - \frac{1}{(r+1)} + \frac{1}{2(r+2)}$ $\Rightarrow \dots \Rightarrow$ $\frac{1}{2} - \frac{1}{2} + \frac{1}{4} \dots + \frac{1}{2(n+1)} - \frac{1}{(n+1)} + \frac{1}{2(n+2)}$ $= \frac{1}{4} - \frac{1}{2} \left\{ \frac{1}{(n+1)(n+2)} \right\} \quad (\text{OE})$ $\therefore \sum_1^\infty \frac{1}{r(r+1)(r+2)} = \frac{1}{4}$	(M1)  (A1)  (A1)  (A1√)	(3)  (1)	

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4	<p>Draws sketch of <math>C</math>.</p> <p>Uses <math>\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta</math></p> <p>Uses double angle formula.</p> <p>Integrates and obtains area.</p> <p>Finds areas.</p>	<p>Shows <math>(4,0)</math> and <math>(0,\pi)</math> lie on <math>C</math>. Correct shape. (Full cardioid is B1 unless clear evidence of plotting up to <math>2\pi</math> or <math>-\pi</math> to <math>\pi</math>.)</p> $\frac{1}{2} \int_0^{\pi} (4 + 8\cos\theta + 4\cos^2\theta) d\theta$ $= \int_0^{\pi} (2 + 4\cos\theta + 2\cos^2\theta) d\theta$ $= \int_0^{\pi} (3 + 4\cos\theta + \cos 2\theta) d\theta$ $= \left[ 3\theta + 4\sin\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi} = 3\pi$ <p>(A1 for correct integral)</p> $\frac{3\pi}{5} + 4\sin\frac{\pi}{5} + \sin\frac{\pi}{5}\cos\frac{\pi}{5} = 4.712$ $3\pi - 4.712 = 4.713$	<p>B1 B1</p> <p>M1</p> <p>M1</p> <p>A1A1 CWO</p> <p>M1A1</p> <p>A1</p>	<p>2</p> <p>4</p> <p>3</p>	<p>[9]</p>
5	<p>Identifies matrices <math>\mathbf{P}</math> and <math>\mathbf{D}</math>.</p> <p>Finds inverse of <math>\mathbf{P}</math>.</p> <p>Uses appropriate result to obtain <math>\mathbf{A}</math>. (First mark can be implied by correct working.)</p>	$\mathbf{P} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & -1 & -3 \\ -2 & 3 & 5 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ <p>Det <math>\mathbf{P} = 1</math></p> $\mathbf{P}^{-1} = \text{Adj } \mathbf{P} = \begin{pmatrix} 4 & 11 & 5 \\ 1 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D} \Rightarrow \mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ $\mathbf{A} = \begin{pmatrix} 0 & -1 & 2 \\ -1 & -1 & -3 \\ -2 & 3 & 5 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 4 & 11 & 5 \\ 1 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ $= \begin{pmatrix} 0 & -1 & 4 \\ -1 & -1 & -6 \\ 2 & 3 & 10 \end{pmatrix} \begin{pmatrix} 4 & 11 & 5 \\ 1 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ $= \begin{pmatrix} 3 & 4 & 2 \\ -11 & -27 & -13 \\ 21 & 54 & 26 \end{pmatrix}$	<p>B1B1</p> <p>B1</p> <p>M1A1</p> <p>M1</p> <p>M1A1√</p> <p>A1</p>	<p>9</p>	<p>[9]</p>
5	<p><b>Alternative Approach:</b></p> $\mathbf{A} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$	<p>Use of <math>\mathbf{Ae} = \lambda\mathbf{e}</math></p> <p>Obtains 3 sets of 3 linear equations: One set Other two sets</p> <p>Solves one set Solves other sets</p>	<p>(M1)</p> <p>(M1A1) (A1A1)</p> <p>(M1A1) (A1A1)</p>	<p>(9)</p>	<p>[9]</p>

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6	Obtains fifth roots of unity by de M's Thm.	$1 = \cos(2k\pi) + i \sin(2k\pi) \quad k = 0, 1, 2, 3, 4.$ $\theta = \left(\frac{2k\pi}{5}\right) \quad k = 0, 1, 2, 3, 4.$	M1	2	
	Rewrites	$(z+1)^5 = z^5 \Rightarrow \frac{(z+1)^5}{z^5} = 1 \Rightarrow \left(\frac{z+1}{z}\right)^5 = 1$	A1		
	and factorises.	$\frac{z+1}{z} = \text{cis}\left(\frac{2k\pi}{5}\right) \Rightarrow z+1 = z \text{cis}\left(\frac{2k\pi}{5}\right)$ $\Rightarrow z\left(1 - \text{cis}\left(\frac{2k\pi}{5}\right)\right) = -1$	M1		
	Isolates z.	$\Rightarrow z = \frac{-1}{1 - \text{cis}\left(\frac{2k\pi}{5}\right)} = \frac{-\left(\text{cis}\left(-\frac{k\pi}{5}\right)\right)}{\text{cis}\left(-\frac{k\pi}{5}\right) - \text{cis}\left(\frac{k\pi}{5}\right)}$	A1		
	Obtains purely imaginary denominator	$= \frac{-\cos\left(\frac{k\pi}{5}\right) + i \sin\left(\frac{k\pi}{5}\right)}{-2i \sin\left(\frac{k\pi}{5}\right)} = -\frac{1}{2} + \frac{1}{2i} \cot\left(\frac{k\pi}{5}\right) \quad k = 1, 2, 3,$	M1A1		
	and obtains result.	$= -\frac{1}{2} \left(1 + i \cot\left(\frac{k\pi}{5}\right)\right) \quad k = 1, 2, 3, 4. \quad (\text{AG})$  Observes that original equation is a quartic with real coefficients, so roots occur in conjugate pairs and $k = 0$ must be rejected.	A1		
			B1	7	[9]

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7	<p>Reduces <math>M_1</math> to echelon form.</p> <p>Finds <math>\text{Dim}(K_1)</math></p> <p>Reduces <math>M_2</math> to echelon form.</p> <p>Finds <math>\text{Dim}(K_2)</math></p> <p>Obtains basis for <math>K_1</math>.</p> <p>Obtains basis for <math>K_2</math>.</p> <p>and shows <math>K_2 \subset K_1</math>.</p>	$\begin{pmatrix} 1 & 1 & 1 & 4 \\ 2 & 1 & 4 & 11 \\ 3 & 4 & 1 & 9 \\ 4 & -3 & 18 & 37 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} 1 & 1 & 1 & 4 \\ 0 & -1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p><math>\text{Dim}(K_1) = 4 - 2 = 2</math> (AG)</p> $\begin{pmatrix} 1 & 1 & 1 & -1 \\ 2 & 3 & 0 & 1 \\ 3 & 4 & 1 & 0 \\ 4 & 5 & 2 & 0 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (\text{aef})$ <p><math>\text{Dim}(K_2) = 4 - 3 = 1</math> (AG)</p> $x + y + z + 4t = 0$ $-y + 2z + 3t = 0$ <p>Legitimately obtains:</p> <p>Basis for <math>K_1</math> is <math>\left\{ \begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -7 \\ 3 \\ 0 \\ 1 \end{pmatrix} \right\}</math> (OE)</p> $x + y + z - t = 0$ $y - 2z + 3t = 0$ $t = 0$ <p>Legitimately obtains:</p> <p>Basis for <math>K_2</math> is <math>\left\{ \begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \end{pmatrix} \right\}</math> (OE) <math>\Rightarrow K_2 \subset K_1</math></p>	<p>M1A1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1</p>	<p>5</p> <p>5</p>	<p>[10]</p>
8	<p>Forms AQE and solves. Writes CF.</p> <p>Correct form for PI and differentiates twice. Substitutes. Writes PI.</p> <p>Writes GS.</p> <p>Uses <math>y(0) = 5</math> to find <math>A</math>. Uses <math>y'(0) = 1</math> to find <math>B</math>.</p> <p>Writes particular solution.</p>	$m^2 + 2m + 5 = 0 \Rightarrow m = -1 \pm 2i$ <p>CF: <math>y = e^{-x} (A \cos 2x + B \sin 2x)</math></p> $y = ke^{-2x} \Rightarrow y' = -2ke^{-2x} \Rightarrow y'' = 4e^{-2x}$ $\Rightarrow 4k - 4k + 5k = 10 \Rightarrow k = 2$ <p>PI: <math>y = 2e^{-2x}</math></p> <p>GS: <math>y = e^{-x} (A \cos 2x + B \sin 2x) + 2e^{-2x}</math></p> $y = 5, x = 0 \Rightarrow 5 = A + 2 \Rightarrow A = 3$ $y' = -e^{-x} (A \cos 2x + B \sin 2x) + e^{-x} (-2A \sin 2x + 2B \cos 2x) - 4e^{-2x}$ $y' = 1, x = 0 \Rightarrow 1 = -3 + 2B - 4 \Rightarrow B = 4$ $\therefore y = e^{-x} (3 \cos 2x + 4 \sin 2x) + 2e^{-2x}$	<p>M1A1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>CAO</p>	<p>11</p>	<p>[11]</p>

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9(i) and (ii)	<p><b>Possible approach for first two parts together.</b></p> <p><b>(i)</b> can come after finding turning points: Continuous function (implied by graph) ⇒ (2,2.5) Max and (-1,1) Min ⇒ <math>1 \leq y \leq \frac{5}{2}</math> (AG) N.B. Award B1 if Max and Min assumed without proof. i.e. 1/4 .</p>	<p>Writes <math>y = \frac{2x^2 + 2x + 3}{x^2 + 2} = 1 + \frac{(x+1)^2}{x^2 + 2}</math></p> <p>States <math>\frac{(x+1)^2}{x^2 + 2} \geq 0 \Rightarrow y \geq 1</math></p> <p>From this it is clear that (-1, 1) is a turning point.</p> <p>Writes <math>y = \frac{2x^2 + 2x + 3}{x^2 + 2} = \frac{5}{2} - \frac{(x-2)^2}{2(x^2 + 2)}</math></p> <p>States <math>\frac{(x-2)^2}{2(x^2 + 2)} \geq 0 \Rightarrow y \leq \frac{5}{2}</math></p> <p>From this it is clear that <math>(2, 2\frac{1}{2})</math> is the other turning point.</p>	<p>(B1)</p> <p>(B1)</p> <p>(M1A1)</p> <p>(B1)</p> <p>(B1)</p> <p>(A1)</p> <p>(M1)</p> <p>(M1A1)</p> <p>(A1)</p>	<p>(7)</p> <p>(4)</p>
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9	Forms quadratic equation in $x$ .	$yx^2 + 2y = 2x^2 + 2x + 3$	M1	4	
	Uses discriminant to obtain condition for real roots.	$\Rightarrow (y - 2)x^2 - 2x + (2y - 3) = 0$ For real $x$ $4 - 4(y - 2)(2y - 3) \geq 0$ $\Rightarrow (2y - 5)(y - 1) \leq 0$ $\Rightarrow 1 \leq y \leq \frac{5}{2}$ (AG)	A1 M1		
	Differentiates and equates to zero.	$y' = 0$ $\Rightarrow (x^2 + 2)(4x + 2) - 2x(2x^2 + 2x + 3) = 0$ $\Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = -1$ or $x = 2$	M1		
	Solves equation.	(Or substitutes $y = 1$ and $\frac{5}{2}$ in equation of C.)			
	States coordinates of turning points.	Turning points are $(-1, 1)$ and $(2, 2\frac{1}{2})$	A1A1		
Expresses $y$ in an appropriate form. (May alternatively divide numerator and denominator by $x^2$ .)	$y = 2 + \frac{2x - 1}{x^2 + 2}$ As $x \rightarrow \pm\infty$ $y \rightarrow 2 \therefore y = 2$	M1 A1	2		
Finds $y$ -intercept and intersection with $y = 2$ .	Shows $(0, 1\frac{1}{2})$ and $(\frac{1}{2}, 2)$	B1			
Completes graph.	Completely correct graph.	B1	2	<b>[11]</b>	

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10	<p>Differentiates and squares.</p> <p>Uses formula for arc length.</p> <p>Integrates and obtains value.</p> <p>Uses formula for x-coordinate of centroid.</p> <p>Integrates both expressions and obtains value.</p> <p>Uses formula for y-coordinate of centroid.</p> <p>Integrates both expressions and obtains value.</p>	$y' = \frac{1}{\sqrt{3}} x^{\frac{1}{2}} \Rightarrow (y')^2 = \frac{x}{3}$ $s = \int_0^3 \sqrt{1 + \frac{x}{3}} dx$ $= \left[ 2 \left( 1 + \frac{x}{3} \right)^{\frac{3}{2}} \right]_0^3 = 4\sqrt{2} - 2 = 2(2\sqrt{2} - 1) \quad (\text{AG})$ $\bar{x} = \frac{\int_0^3 \frac{2}{3\sqrt{3}} x^{\frac{5}{2}} dx}{\int_0^3 \frac{2}{3\sqrt{3}} x^{\frac{3}{2}} dx}$ $= \frac{\left[ \frac{2}{7} x^{\frac{7}{2}} \right]_0^3}{\left[ \frac{2}{5} x^{\frac{5}{2}} \right]_0^3} = \frac{15}{7} \quad (= 2.14)$ $\bar{y} = \frac{\int_0^3 \frac{1}{2} \times \frac{4}{27} x^3 dx}{\int_0^3 \frac{2}{3\sqrt{3}} x^{\frac{3}{2}} dx}$ $= \frac{\frac{2}{27} \left[ \frac{x^4}{4} \right]_0^3}{\frac{2}{3\sqrt{3}} \left[ \frac{2}{5} x^{\frac{5}{2}} \right]_0^3} = \frac{5}{8} \quad (= 0.625)$	<p>B1</p> <p>M1</p> <p>A1A1</p> <p>M1</p> <p>A1A1 A1</p> <p>M1</p> <p>A1 A1</p>	<p>4</p> <p>7</p>	<p>[11]</p>
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11	<p><b>EITHER</b></p> <p>Note:</p> <p>(1) the parts can be either way round.</p> <p>(2) Insertion of limits in <math>[e^x \sin x]</math> causes the term to vanish.</p>	$I = \int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx$ $= -e^x \cos x + e^x \sin x - \int e^x \sin x dx$ $\therefore 2I = e^x(\sin x - \cos x)$ $\therefore \int_0^\pi e^x \sin x dx = \left[ \frac{1}{2} e^x(\sin x - \cos x) \right]_0^\pi$ $= \frac{e^\pi}{2} - \left( -\frac{1}{2} \right) = \frac{1+e^\pi}{2} \quad (\text{AG})$	M1 A1 M1	4	
		$I_n = \int_0^\pi e^x \sin^n x dx$ $= \left[ \sin^n x \cdot e^x \right]_0^\pi - \int_0^\pi e^x (n \sin^{n-1} x \cos x) dx$ $= \left\{ \begin{array}{l} 0 - \left[ n \sin^{n-1} x \cos x \cdot e^x \right]_0^\pi \\ + n \int_0^\pi e^x (\cos^2 x (n-1) \sin^{n-2} x - \sin^{n-1} x \sin x) dx \end{array} \right\}$ $= 0 + n(n-1) \int_0^\pi e^x \cos^2 x \sin^{n-2} x dx - nI_n \quad (\text{AG})$ $= n(n-1) \int_0^\pi e^x (1 - \sin^2 x) \sin^{n-2} x dx - nI_n$ $\therefore (n+1)I_n = n(n-1)I_{n-2} - n(n-1)I_n$ $\therefore (n(n-1) + n+1)I_n = n(n-1)I_{n-2}$ $\therefore (n^2 + 1)I_n = n(n-1)I_{n-2} \quad (\text{AG})$	M1 A1 A1 M1A1		
$I_5 = \frac{20}{26} I_3 = \frac{20}{26} \times \frac{6}{10} I_1$ $\Rightarrow I_5 = \frac{6}{13} \times \left( \frac{1+e^\pi}{2} \right) = \frac{3}{13} (1+e^\pi)$	M1 A1	4	[14]		
$\text{Mean value} = \frac{\int_0^\pi e^x \sin^5 x dx}{\pi - 0} = \frac{3}{13\pi} (1+e^\pi)$	M1A1				

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11	OR	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ 0 & 1 & m-1 \end{vmatrix} = -m\mathbf{i} + 4(1-m)\mathbf{j} + 4\mathbf{k}$ $\frac{\begin{vmatrix} 1 & -m \\ 0 & 4-4m \\ -4 & 4 \end{vmatrix}}{\sqrt{m^2 + 16(1-2m+m^2) + 16}} = 3$ $\Rightarrow \dots \Rightarrow 19m^2 - 40m + 4 = 0$ $\Rightarrow (19m-2)(m-2) = 0$ $\Rightarrow m = 2, \text{ since } m \text{ is an integer. (AG)}$	M1A1	7	[14]	
	Obtains direction of common perpendicular.					
	Uses result for shortest distance between lines.					M1A1
	Solves equation.					A1
						M1
						A1
	Finds relevant vectors.		$\mathbf{CA} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} \text{ and } \mathbf{CD} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \text{ or } \mathbf{AD} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$			B1
Use of cross-product.	$\frac{1}{\sqrt{17}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ 1 & 0 & -4 \end{vmatrix} = \frac{1}{\sqrt{17}} \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix}$	M1				
Obtains shortest distance.	$\frac{1}{\sqrt{17}} \sqrt{4^2 + 1^2 + 1^2} = \sqrt{\frac{18}{17}} \quad (= 1.03)$	A1	3			
Finds 2 <sup>nd</sup> vector in BCD (CD may already have been found.)	$\mathbf{BC} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$	B1				
Finds normal vector to BCD. (Normal to ACD already found.)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 0 & 1 & 1 \end{vmatrix} = -6\mathbf{i} - 3\mathbf{j} + 3\mathbf{k} \sim 2\mathbf{i} + \mathbf{j} - \mathbf{k}$	M1				
Finds angle between planes = angle between normal vectors.	$\cos \theta = \frac{(4\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k})}{\sqrt{16+1+1}\sqrt{4+1+1}} = \frac{6}{\sqrt{18}\sqrt{6}} = \frac{1}{\sqrt{3}}$ $\therefore \text{Angle between planes} = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \text{ (AG)}$	M1	A1	4		

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11	<p><b>OR</b> <b>Alternatives for middle part:</b></p> <p>Or (a) Vector from <math>D</math> to any point on <math>AC</math></p> <p>Uses orthogonality to obtain <math>t</math>.</p> <p>Finds magnitude of perpendicular.</p> <p>Or (b) Finds length of <math>AD</math> (or <math>CD</math>)</p> <p>Finds projection of <math>AD</math> (or <math>CD</math>) onto <math>AC</math>.</p> <p>Finds perpendicular by Pythagoras.</p>	$\begin{pmatrix} 1+t \\ -1 \\ -5-4t \end{pmatrix}$ $\begin{pmatrix} 1+t \\ -1 \\ -5-4t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} = 0 \Rightarrow t = -\frac{21}{17}$ $\frac{1}{\sqrt{17}} \sqrt{4^2 + 1^2 + 1^2} = \sqrt{\frac{18}{17}} \quad (= 1.03)$ $ \overrightarrow{AD}  = \sqrt{27}$ $\frac{\begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}}{\sqrt{4^2 + 1^2}} = \frac{21}{\sqrt{17}}$ $\sqrt{27 - \frac{441}{17}} = \sqrt{\frac{18}{17}} \quad (= 1.03)$	(B1)	(M1)	(A1)	(3)
			(B1)			(M1)
			(A1)			(3)