

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

Cambridge International Advanced Level

## **MARK SCHEME for the May/June 2015 series**

### **9231 FURTHER MATHEMATICS**

**9231/22**

Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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## **Mark Scheme Notes**

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\Rightarrow$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

|               |                                                        |                 |              |
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The following abbreviations may be used in a mark scheme or used on the scripts:

|     |                                                                                                                                                                                         |
|-----|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF | Any Equivalent Form (of answer is equally acceptable)                                                                                                                                   |
| AG  | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                     |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)                                                                                                  |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)                                                                                             |
| CWO | Correct Working Only – often written by a "fortuitous" answer                                                                                                                           |
| ISW | Ignore Subsequent Working                                                                                                                                                               |
| MR  | Misread                                                                                                                                                                                 |
| PA  | Premature Approximation (resulting in basically correct work that is insufficiently accurate)                                                                                           |
| SOS | See Other Solution (the candidate makes a better attempt at the same question)                                                                                                          |
| SR  | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

### **Penalties**

|       |                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.                                                                                                                                                                                                                                                                                                                          |

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| Question Number | Mark Scheme Details                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Part Mark | Total    |
|-----------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------|
| <b>1</b>        | Find $T$ by equating $dv/dt$ at $t = T$ to 6: $4T - 4 = 6$ , $T = 2.5$ M1 A1<br>Find radial component $v^2/r$ of acceln. at $t = T$ : $v^2/r = (2T^2 - 4T + 3)^2 / 0.25$<br>(M0 if $T$ not given a value) $= (11/2)^2 \times 4 = 121 \text{ [m s}^{-2}\text{]}$ M1 A1<br><b>SR:</b> Max M1 (1/4) if linear and angular confused                                                                                                                                                                                                                                                                                                                                                                                                                                           | 4         | <b>4</b> |
| <b>2 (i)</b>    | Find $\omega^2$ from SHM eqn. $d^2x/dt^2 = -\omega^2x$ at $C$ : $0.625 = 10\omega^2$ , $\omega^2 = 0.0625$ or $1/16$ B1<br>Find period $T$ [s] from $T = 2\pi/\omega$ : (ft on $\omega^2$ ) $T = 2\pi/\sqrt{1/16} = 8\pi$ (not 25.1) B1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | 2         | <b>8</b> |
| <b>(ii)</b>     | Find amplitude $a$ [m] from $v_C^2 = \omega^2(a^2 - 10^2)$ : $6^2 = \omega^2(a^2 - 10^2)$<br>Find time from $C$ to $M$ , e.g.: $a^2 = 6^2 \times 16 + 10^2$ , $a = \sqrt{676} = 26$ M1 A1<br>(AEF throughout) $\omega^{-1} \sin^{-1}(10/a) + \omega^{-1} \sin^{-1} 1/2$ or<br>$\omega^{-1} \cos^{-1}(-10/a) - \omega^{-1} \cos^{-1} 1/2$ or<br>$1/2T - \omega^{-1} \cos^{-1}(10/a) - \omega^{-1} \cos^{-1} 1/2$ M1<br>$= \omega^{-1} \{0.3948 + \pi/6 [= 0.5236]\}$ or<br>$\omega^{-1} \{1.9656 - \pi/3 [= 1.0472]\}$ or<br>$\omega^{-1} \{\pi - 1.760 - \pi/3 [= 1.0472]\}$ A1<br>$= 1.579 + 2.094$ or $7.862 - 4.189$<br>or $12.567 - 4.704 - 4.189$<br>or $4 \times 0.9184$ ; $= 3.67$ [s] A1; A1                                                                      | 4         |          |
| <b>3</b>        | Find $v^2$ from conservation of energy: $1/2mv^2 = 1/2mu^2 + mga(1 - \cos \theta)$ M1 A1<br>Find $R$ by using $F = ma$ radially: $R = mg \cos \theta - mv^2/a$ B1<br>Eliminate $v^2$ to find $R$ : AG $R = mg(3 \cos \theta - 2) - mu^2/a$ M1 A1<br>Find $u^2$ or $v^2$ in terms of $\cos \theta$ when $R = 0$ : $u^2 = ag(3 \cos \theta - 2)$ or<br>$v^2 = ag \cos \theta$ B1<br><i>EITHER:</i> Replace $\cos \theta$ in energy eqn with $v = 2u$ : $4u^2 = u^2 + 2ag - 2/3(u^2 + 2ag)$<br>or $u^2 + 2ag - 8u^2$ M1 A1<br><i>OR:</i> Find $\cos \theta$ and substitute in energy eqn: $[v^2/ag =] 4(3 \cos \theta - 2) = \cos \theta$<br>$\cos \theta = 8/11$<br>$4u^2 = u^2 + 2ag(1 - 8/11)$ (M1 A1)<br>Hence find $u$ : $u = \sqrt{(2ag/11)}$ or $0.426\sqrt{(ag)}$ A1 | 5         | <b>9</b> |
| <b>4 (i)</b>    | Take moments for rod about $B$ : $W \times a \cos 30^\circ + 3W \times 2a \cos 30^\circ$<br>(or with $\cos 30^\circ = \sqrt{3}/2$ ) $= T \times 2a \cos 30^\circ$ M1 A1<br>Hence find tension $T$ : $T = 7W/2$ A1<br>(Can earn M1 A0 A1 if e.g. $\sin 30^\circ$ wrongly used)<br>Find modulus $\lambda$ using Hooke's Law: $T = \lambda(2a - 3a/5) / (3a/5)$<br>$\lambda = (3/7)(7W/2) = 3W/2$ M1 A1                                                                                                                                                                                                                                                                                                                                                                      | 5         |          |

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| Question Number | Mark Scheme Details                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Part Mark | Total |
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| (ii)            | <p><i>EITHER:</i> Find horizontal component of force <math>F</math> at <math>B</math>: <math>X = T \cos 30^\circ</math><br/> <math>= (7\sqrt{3}/4) W</math> or <math>3.03 W</math> B1✓</p> <p>Find vertical component: (<math>X, Y</math> ft on <math>T</math>) <math>Y = 4W - T \sin 30^\circ = 9W/4</math> B1✓</p> <p>Find magnitude of <math>F</math>: <math>F = \sqrt{X^2 + Y^2}</math><br/> <math>= (\sqrt{57}/2) W</math> or <math>3.77[5] W</math> B1</p> <p>Find direction of <math>F</math> (AEF): Upward force at angle to <math>AB</math> of<br/> <math>\tan^{-1} Y/X = \tan^{-1} 3\sqrt{3}/7</math><br/> (A0 if direction unclear) <math>= 36.6^\circ</math> or <math>0.639</math> radians M1 A1</p> <p><i>OR:</i> Find component along <math>CB</math>: <math>F_1 = (4W + T) \sin 30^\circ = 15W/4</math> (B1✓)</p> <p>Find normal component: (<math>F_1, F_2</math> ft on <math>T</math>) <math>F_2 = (4W - T) \cos 30^\circ = (\sqrt{3}/4) W</math> (B1✓)</p> <p>Find magnitude of <math>F</math>: <math>F = (\sqrt{57}/2) W</math> or <math>3.77[5] W</math> (B1)</p> <p>Find direction of <math>F</math> (AEF): Upward force at angle to <math>CB</math> of<br/> <math>\tan^{-1} F_2/F_1 = \tan^{-1} \sqrt{3}/15</math><br/> (A0 if direction unclear) <math>= 6.6^\circ</math> or <math>0.115</math> radians (M1 A1)</p> <p><i>OR:</i> Find component parallel to string <math>CA</math>: <math>\pm F_1 = T - 4W \sin 30^\circ = 3W/2</math> (B1✓)</p> <p>Find normal component: (<math>F_1, F_2</math> ft on <math>T</math>) <math>\pm F_2 = 4W \cos 30^\circ = 2\sqrt{3} W</math> (B1✓)</p> <p>Find magnitude of <math>F</math>: <math>F = (\sqrt{57}/2) W</math> or <math>3.77[5] W</math> (B1)</p> <p>Find direction of <math>F</math> (AEF): Upward force at angle to <math>AC</math> of<br/> <math>\tan^{-1} F_2/F_1 = \tan^{-1} 4/\sqrt{3}</math><br/> (A0 if direction unclear) <math>= 66.6^\circ</math> or <math>1.16</math> radians (M1 A1)</p> | 5         | 10    |
| 5               | <p>For <math>A</math> &amp; <math>B</math> use conservation of momentum, e.g.: <math>3mv_A + 2mv_B = 3mu</math> M1</p> <p>(<math>m</math> may be omitted here and below)</p> <p>Use Newton's law of restitution (consistent signs): <math>v_B - v_A = eu</math> M1</p> <p>Combine to find <math>v_B</math>: <math>v_B = 3(1 + e) u/5</math> A1</p> <p>For <math>B</math> &amp; <math>C</math> use conservation of momentum, e.g.: <math>2mv_B' + mv_C = 2mv_B</math> M1</p> <p>Use Newton's law of restitution (consistent signs): <math>v_C - v_B' = e' v_B</math> M1</p> <p>Combine to find <math>v_C</math> and <math>v_B'</math>: <math>v_C = 2(1 + e') v_B/3</math><br/> <math>= 2(1 + e)(1 + e') u/5</math> AG A1</p> <p><math>v_B' = (2 - e') v_B/3</math><br/> <math>= (1 + e)(2 - e') u/5</math> A1</p> <p>Find ratios or values of <math>v_A, v_B', v_C</math> from momentum: <math>3v_A = 2v_B' = v_C [= u]</math> B1</p> <p>Find <math>e</math> from first collision eqns, e.g.: <math>v_A = (3 - 2e) u/5 = u/3</math><br/> (or find <math>e'</math> and then use <math>3v_A = 2v_B'</math>)<br/> <math>\text{or } v_B = \frac{1}{2}(3u - u) \text{ or } (\frac{1}{3} + e) u</math><br/> <math>= 3(1 + e) u/5, e = \frac{2}{3}</math> M1 A1</p> <p>Find <math>e'</math> from second collision eqns, e.g.: <math>2v_B' = v_C</math> so <math>2(2 - e') = 2(1 + e')</math><br/> <math>\text{or } v_C = 2(1 + \frac{2}{3})(1 + e') u/5 = u</math><br/> <math>\text{or } v_B' = (1 + \frac{2}{3})(2 - e') u/5 = u/2</math><br/> <math>e' = \frac{1}{2}</math> M1 A1</p>                                                                                                                                                                                                                                                                                                                                                                                              | 7         | 12    |
| 6               | <p>Equate pooled estimate of <math>\sigma^2</math> to 12: <math>(11 - 5^2/N + 160 - 10^2/10) /</math><br/> <math>(N + 10 - 2) = 12</math> M1 A1</p> <p>Formulate and solve relevant quadratic eqn. for <math>N</math>: <math>12N^2 - 65N + 25 = 0, N = 5</math> M1 A1</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    | 4         | 4     |

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| 7               | <p>Find <math>\Sigma x</math> via sample mean <math>\bar{x}</math>: <math>\Sigma x = 8\bar{x} = 8 \times \frac{1}{2}(1.17 + 2.03) = 8 \times 1.6 = 12.8</math> M1 A1</p> <p>Find estimate of population variance <math>s^2</math>: <math>t \sqrt{(s^2/8)} = \frac{1}{2}(2.03 - 1.17) [= 0.43]</math> M1</p> <p>Use of correct tabular value (1.96 leads to 23.2): <math>t_{7, 0.975} = 2.36[5]</math> A1</p> <p>(to 3 d.p.) <math>s^2 = 0.2645</math> or <math>32/121</math> or <math>0.5143^2</math> A1</p> <p>Find <math>\Sigma x^2</math> from <math>s^2</math>: <math>s^2 = \{\Sigma x^2 - (\Sigma x)^2/8\}/7</math></p> <p>(M0 for <math>s^2 = \{\dots\}/8</math>) <math>\Sigma x^2 = 7 \times 0.2645 + 12.8^2/8 = 22.3</math> M1 A1</p> | 7         | 7     |
| 8 (a) (i)       | Find correlation coefficient $r$ from $r^2 = b_1 b_2$ : $r = \sqrt{(0.38 \times 0.96)} = 0.604$ M1 *A1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 2         | 8     |
| (ii)            | <p>State both hypotheses (B0 for <math>r \dots</math>): <math>H_0: \rho = 0, H_1: \rho &gt; 0</math> B1</p> <p>State or use correct tabular one-tail <math>r</math>-value: <math>r_{10, 5\%} = 0.549</math> *B1</p> <p>State or imply valid method for reaching conclusion: Reject <math>H_0</math> if <math> r  &gt; \text{tab. value}</math> (AEF) M1</p> <p>Correct conclusion (AEF, dep *A1, *B1): There is positive correlation A1</p>                                                                                                                                                                                                                                                                                                   | 4         |       |
| (b)             | <p>State or use relevant tabular two-tail <math>r</math>-value: <math>r_{16, 5\%} = 0.497</math> (or <math>r_{15, 5\%} = 0.514</math>) M1</p> <p>Find least possible value of <math>n</math>: <math>n_{\min} = 16</math> A1</p> <p>SR M1 A1 for stating 16 without explanation</p> <p>B1 for stating 15 without explanation</p> <p>B1 for finding or stating one-tail result 12</p>                                                                                                                                                                                                                                                                                                                                                           | 2         |       |
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |           |       |
| 9 (i)           | <p>Relate <math>P(X &gt; x)</math> to number of flaws (AEF): <math>P(X &gt; x) = P(\text{zero flaws in } x \text{ m})</math> B1</p> <p>Relate this to Poisson distn. (AEF): <math>= P_0(0.8x) = e^{-0.8x}</math> A.G. B1</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 2         | 11    |
| (ii)            | <p>Find <math>P(\text{number of flaws} \geq 1)</math>: <math>1 - P_0(0.8 \times 4) = 1 - e^{-3.2}</math></p> <p>(M0 if “1 – “ omitted) <math>= 1 - 0.0408 = 0.959</math> M1 A1</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | 2         |       |
| (iii) (a)       | Find or state distribution function $F(x)$ : $F(x) = P(X \leq x) = 1 - P(X > x) = 1 - e^{-0.8x}$ B1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 1         |       |
| (b)             | Find or state probability density function $f(x)$ : $f(x) = dF/dx = 0.8 e^{-0.8x}$ M1 A1                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      | 2         |       |
| (c)             | <p>Formulate equation for either quartile value <math>Q</math>: <math>F(Q) = 1 - e^{-0.8Q} = \frac{1}{4}</math> or <math>\frac{3}{4}</math> M1</p> <p>Find lower quartile <math>Q_1</math>: (AEF) <math>Q_1 = 1.2 \ln 4/3 [= 0.360]</math> A1</p> <p>Find upper quartile <math>Q_3</math>: (AEF) <math>Q_3 = 1.2 \ln 4 [= 1.733]</math> A1</p> <p>Find interquartile range (allow <math>Q_1 - Q_3</math>): <math>Q_3 - Q_1 [= 1.2 \ln 3] = 1.37</math> A1</p>                                                                                                                                                                                                                                                                                 | 4         |       |

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| 10              | <p>Calculate gradient <math>b_1</math> in <math>y - \bar{y} = b_1(x - \bar{x})</math> : <math>S_{xy} = 313.28 - 50.8 \times 56.9/10</math><br/> <math>= 24.228</math><br/> <math>S_{xx} = 284.16 - 50.8^2/10 = 26.096</math><br/> (PA –1 so max 4/5 for 0.93, giving <math>y = 7.48</math>) <math>b_1 = S_{xy} / S_{xx} = 0.928</math> M1 A1<br/> Find <math>y</math> when <math>x = 7</math> from regression line of <math>y</math> on <math>x</math>: <math>y = 56.9/10 + b_1(7 - 50.8/10)</math> M1 A1<br/> <math>= 5.69 + 0.928(7 - 5.08)</math><br/> <math>[ = 0.928 \times 7 + 0.976 ]</math><br/> <math>y = 7.47</math> (allow 7.48 or 7.5) A1<br/> SR If regression line of <math>x</math> on <math>y</math> used: <math>S_{yy} = 347.59 - 56.9^2/10 = 23.829</math><br/> <math>b_2 = S_{xy} / S_{yy} = 1.017</math> (M1)<br/> <math>7 = 50.8/10 + b_2(y - 56.9/10)</math> (M1 A1)<br/> <math>= 1.017y - 0.707</math><br/> (can earn at most 4/5) <math>y = 7.58</math> (allow 7.6) (A1)<br/> Find differences (e.g. <math>y - x</math>) and sample mean: 1 0.7 1.3 0.2 0.1 0.9 0.8 0.5 0.1 0.5<br/> <math>\bar{d} = 6.1 / 10 = 0.61</math> M1 A1<br/> Estimate population variance (to 3 s.f.): <math>s^2 = (5.19 - 6.1^2/10) / 9</math><br/> (allow biased here: 0.1469 or 0.3833<sup>2</sup>) <math>= 0.1632</math> or <math>0.404^2</math> B1<br/> State hypotheses (AEF; B0 for <math>\bar{x}</math>), e.g.: <math>H_0: \mu_y - \mu_x = 0.4</math>, <math>H_1: \mu_y - \mu_x &gt; 0.4</math> B1<br/> Calculate value of <math>t</math>: <math>t = (\bar{d} - 0.4)/(s/\sqrt{10}) = 1.64</math> M1 A1<br/> State or use correct tabular <math>t</math>-value: <math>t_{9, 0.95} = 1.83[3]</math> B1<br/> (or can compare <math>\bar{d}</math> with 0.634)<br/> Consistent conclusion (AEF, ft on both <math>t</math>-values): [Accept <math>H_0</math>:]<br/> No improvement of more than 0.4 B1✓<br/> Wrong test can earn only B1 for hypotheses<br/> and B1 for conclusion</p> | 5         | 13    |
|                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 8         |       |

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| 11A                                                                                    | Find MI of disc about $O$ :                                                                                                                 | $I_{\text{disc}} = \frac{1}{2} 2ma^2 = ma^2$                                                                                | B1          | 4         |       |    |  |
|                                                                                        | Find MI of ring about $O$ :                                                                                                                 | $I_{\text{ring}} = 2m (3a)^2 = 18 ma^2$                                                                                     | B1          |           |       |    |  |
|                                                                                        | <i>EITHER</i> : Find MI of any rod about $O$ :                                                                                              | $I_{\text{rod}} = \frac{1}{3} (3m/2)a^2 + (3m/2)(2a)^2$<br>$= (13/2) ma^2$                                                  | B1          |           |       |    |  |
|                                                                                        | <i>OR</i> : Find MI of 2 collinear rods about $O$ :                                                                                         | $\frac{1}{3} (9m/2)(3a)^2 - \frac{1}{3} (3m/2)a^2$<br>$= 13 ma^2$                                                           | (B1)        |           |       |    |  |
|                                                                                        | Find MI of object about $O$ :                                                                                                               | $I_O = ma^2 + 18 ma^2 + 4(13/2) ma^2$<br>$= 45 ma^2$ AG                                                                     | B1          |           |       |    |  |
|                                                                                        | Find MI of object about axis at $O$ // to tangent:                                                                                          | $I_{O'} = \frac{1}{2}I_O$                                                                                                   | M1          |           |       |    |  |
|                                                                                        | Find MI of object about tangential axis at $A$ :                                                                                            | $I_A = I_{O'} + 10m (3a)^2$<br>$= (225/2) ma^2$                                                                             | M1 A1       |           |       |    |  |
|                                                                                        | Find new MI when particle is attached at $C$ :                                                                                              | $I_A' = I_A + 3m (6a)^2$<br>$= (441/2) ma^2$                                                                                | M1          |           |       |    |  |
|                                                                                        | Find and use initial angular speed:                                                                                                         | $\omega_0 = u/3a$                                                                                                           | B1          |           |       |    |  |
|                                                                                        | Find gain in P.E. at instantaneous rest:                                                                                                    | $(10mg \times 3a + 3mg \times 6a)(1 - \sin \theta)$<br>$= 45mga/2 + 27mga/2$<br><i>or</i> $48mga (1 - \sin \theta) = 36mga$ | M1 A1<br>A1 |           |       |    |  |
| Find $u$ by equating to rotational KE:                                                 | $= \frac{1}{2} I_A' \omega_0^2$<br>$u^2 = (36/441) \times 36ag$ [441=21 <sup>2</sup> ]<br>$u = (12/7) \sqrt{ag}$ <i>or</i> $1.71 \sqrt{ag}$ | M1<br>A1                                                                                                                    | 7           | 14        |       |    |  |
| SR: Taking $AC$ at $\sin^{-1}(1/4)$ to vertical:                                       | P.E. = $48mga (1 - \cos \sin^{-1}(1/4))$<br>$= 1.524 mga$                                                                                   | (A0)                                                                                                                        |             |           |       |    |  |
| (max 6/7)<br>SR: Overlooking added particle can earn<br>M0 B1 M1 A0 A0 M1 A0 (max 3/7) | $u = 0.35[3] \sqrt{ag}$                                                                                                                     | (A1)                                                                                                                        |             |           |       |    |  |
| 11B                                                                                    | State suitable distribution:                                                                                                                | Geometric                                                                                                                   | B1          | 1         |       |    |  |
|                                                                                        | State (at least) null hypothesis: (AEF)<br>(B0 for “It is a good fit”)                                                                      | $H_0$ : Distn. fits data <i>or</i> $p = 0.6$                                                                                | B1          |           |       |    |  |
|                                                                                        | Find exp. values using $200pq^{x-1}$ with $p = 0.6, q = 0.4$ :<br>(ignore incorrect final value here,<br>e.g. 0.4915 which can earn max 5/8 | 120 48 19.2 7.68<br>3.072 1.2288 0.8192                                                                                     | M1          |           |       |    |  |
|                                                                                        | Combine last 3 cells since exp. value < 5:                                                                                                  | $O: \dots 6$<br>$E: \dots 5.12$                                                                                             | B1          |           |       |    |  |
|                                                                                        | Calculate $\chi^2$ (result correct to 3 s.f.):                                                                                              | $\chi^2 = 0.3 + 0.5208 + 0.4083$<br>$+ 2.8519 + 0.15125 = 4.23[2]$                                                          | M1 A1       |           |       |    |  |
|                                                                                        | State or use consistent tabular value (to 3 s.f.):<br>[or if 2 or no cells combined:                                                        | 5 cells: $\chi_{4,0.95}^2 = 9.488$<br>6 cells: $\chi_{5,0.95}^2 = 11.07$<br>7 cells: $\chi_{6,0.95}^2 = 12.59$              | B1✓         |           |       |    |  |
|                                                                                        | or if 4 cells combined, as with 0.4915:                                                                                                     | 4 cells: $\chi_{3,0.95}^2 = 7.815$ ]                                                                                        |             |           |       |    |  |
|                                                                                        | Valid method for reaching conclusion:                                                                                                       | Accept $H_0$ if $\chi^2 < \text{tabular value}$                                                                             |             |           |       | M1 |  |
|                                                                                        | Conclusion (AEF, requires both values correct):<br>(Allow A1 for “It is a good fit”)                                                        | $4.23 < 9.49$ so distn fits <i>or</i> $p = 0.6$                                                                             | A1          |           |       | 8  |  |
|                                                                                        | Find prob. $p$ of at least one 6 on 5 throws of one die:                                                                                    | $p = 1 - 0.75^5 = 0.7627$ (4 s.f.)                                                                                          | M1 A1       |           |       |    |  |
| Find prob. of at least one 6 on exactly 4 of 10 dice:                                  | ${}^{10}C_4 \times p^4 (1 - p)^6$<br>$= 210 \times 6.043 \times 10^{-5}$<br>$= 0.0126$ <i>or</i> $0.0127$                                   | M1; M1<br>A1                                                                                                                |             |           |       |    |  |
|                                                                                        |                                                                                                                                             |                                                                                                                             |             | 5         | 14    |    |  |