CAMBRIDGE
International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/41**

Paper 4 Further Probability & Statistics **May/June 2026**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

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General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: • the specific **content** defined in the mark scheme • the specific **skills** defined in the mark scheme or in the level descriptions • the **standard of response** required as exemplified by the standardisation scripts.
2	Examiners must award **whole marks** (not half marks, or other fractions).
3	Examiners must award marks **positively**. This means that: • marks are not deducted for errors or omissions • marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate • the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind.

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Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
FT	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
ISW	Ignore subsequent work
J	Judgement
JU	Judgement
M0	Method mark awarded zero
M1	Method mark awarded one
M2	Method mark awarded two
MR	Misread
O	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
PE	Judgment made by the PE
Pre	Premature approximation
SC	Special case

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Annotation	Meaning
SEEN	Indicates that work/page has been seen
SF	Error in number of significant figures
	Correct
TE	Transcription error
XP	Correct answer from incorrect working

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Question	Answer	Marks	Guidance
1(a)	Data has outliers / is skewed / lacks symmetry, suggesting population / distribution may not be symmetrical.	**B1**	Condone no explicit mention of population/distribution or a definite statement relating to population. Allow comment about tied ranks.
		1	

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Question	Answer	Marks	Guidance
1(b)	H_0: population median is 8. H_1: population median is greater than 8.	**B1**	Allow *m*, but not μ. If *m* defined, must see **population.** Median, not mean or average.
	$X \sim \mathrm{B}(15,0.5)$	**B1**	SOI, may be implied by $\mathrm{N}(7.5,3.75)$.
	$\mathrm{P}(X \leqslant 3) = (0.5)^{15} + \binom{15}{1}(0.5)^{15} + \binom{15}{2}(0.5)^{15} + \binom{15}{3}(0.5)^{15}$ OR $\pm\left(\frac{3(\pm 0.5) - 7.5}{\sqrt{3.75}}\right)$	**M1**	Attempt at using binomial distribution with 4 terms. May calculate $\mathrm{P}(X \geqslant 12)$. Implied by AWRT 0.018. OR Attempt to use correct normal approximation, ignoring continuity correction. Implied by AWRT $\pm 2.07, \pm 2.32$ or ± 2.58.
	0.0176 OR ± 2.07	**A1**	AWRT 0.018 M1A1 for unsupported 0.018 or ± 2.07 from normal approximation.
	'0.0176' < 0.05, reject H_0 / significant. OR 2.07 > 1.645, reject H_0 / significant.	**M1**	Compare *their* 0.0176 with 0.05 and appropriate conclusion (may be in terms of H_1). *Their* 0.0176 must come from use of binomial. Test result can be implied by an attempt at an appropriate conclusion in context. For normal approximation, compare *their* ± 2.07 (must come from attempt at standardising) with 1.645 or 1.64 or 1.65, signs must be consistent.
	Sufficient evidence to support the manager's belief.	**A1**	Correct conclusion from correct working, ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Do not accept statements such as 'there is insufficient evidence to suggest…'.
		6	

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<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">2(a)</td><td>H_0: reliability of service is independent of bus company.
H_1: reliability of service is not independent of bus company.</td><td>B1</td><td>Allow dependent, association or relationship for not independent, for example,
“H_0: No association/relationship between…,
H_1: Association/relationship between…”.
Must refer to ‘reliability’ and ‘company’ in H_0.</td></tr>
<tr><td><table><tr><td>7 (12.25)</td><td>22 (19.11)</td><td>20 (17.64)</td></tr><tr><td>14 (9.25)</td><td>11 (14.43)</td><td>12 (13.32)</td></tr><tr><td>4 (3.50)</td><td>6 (5.46)</td><td>4 (5.04)</td></tr></table></td><td>M1</td><td>Calculate expected frequencies (must be seen).
At least two correct to at least 2 significant figures.</td></tr>
<tr><td></td><td>A1</td><td>All expected frequencies correct to at least 3 significant figures.</td></tr>
<tr><td><table><tr><td>7 (12.25)</td><td>22 (19.11)</td><td>20 (17.64)</td></tr><tr><td>18 (12.75)</td><td>17 (19.89)</td><td>16 (18.36)</td></tr></table>OR<table><tr><td>29 (31.36)</td><td>20 (17.64)</td></tr><tr><td>25 (23.68)</td><td>12 (13.32)</td></tr><tr><td>10 (8.96)</td><td>4 (5.04)</td></tr></table></td><td>M1</td><td>Combine rows (for example rows 2 and 3).
OR
Combine columns (for example columns A and B)</td></tr>
</table>

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Question	Answer	Marks	Guidance
2(a)	Combining rows 2 and 3: $\frac{(7-12.25)^2}{12.25}+\frac{(22-19.11)^2}{19.11}+\ldots+\frac{(16-18.36)^2}{18.36}$ $[=2.25+0.43705+0.31574+2.16176+0.41991+0.30336]$ OR $\frac{7^2}{12.25}+\frac{22^2}{19.11}+\ldots+\frac{16^2}{18.36}-100$ Combining columns *A* and *B*: $\frac{(29-31.36)^2}{31.36}+\frac{(20-17.64)^2}{17.64}+\ldots+\frac{(4-5.04)^2}{5.04}$ $[=0.17760+0.31574+0.07358+0.13081+0.12071+0.21460]$ OR $\frac{29^2}{31.36}+\frac{20^2}{17.64}+\ldots+\frac{4^2}{5.04}-100$ No rows / columns combined: $\frac{(7-12.25)^2}{12.25}+\frac{(22-19.11)^2}{19.11}+\ldots+\frac{(4-5.04)^2}{5.04}$ $[=2.250+0.43705+0.31574+2.4392+0.81531+0.13081+0.07143+0.05341+0.21460]$	**M1**	At least 2 expressions (not including 100) of correct form seen. May see individual terms not summed. May be implied by 2 contributions correct to 3 significant figures seen. May be implied by AWRT 5.89 or AWRT 1.03 or AWRT 6.73.
	Combining rows 2 and 3: Test statistic = 5.888 OR Combining columns *A* and *B*: Test statistic = 1.033	**A1**	AWRT 5.89 OR AWRT 1.03

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Question	Answer	Marks	Guidance
2(a)	Combining rows 2 and 3: '5.888' < 5.991, accept H_0 / do not reject H_0 / not significant. OR Combining columns *A* and *B*: '1.033' < 5.991, accept H_0 / do not reject H_0 / not significant. OR No cells combined: '6.73' < 9.488, accept H_0 / do not reject H_0 / not significant.	**M1**	Compare *their* test statistic with 5.991 (or 9.488 if no rows/columns combined) and appropriate conclusion (may be in terms of H_1). Test result can be implied by an attempt at an appropriate conclusion in context ignoring hypotheses.
	Insufficient evidence to suggest reliability (of service) is not independent of (bus) company. OR Insufficient evidence to suggest reliability (of service) is associated with (bus) company. OR Insufficient evidence to suggest that reliability (of service) is dependent on (bus) company.	**A1**	Correct conclusion from correct working ignoring hypotheses. In context. Must refer to both reliability and company. Level of uncertainty in language is used (for example not 'prove'). Allow 'Not enough evidence / no sufficient evidence that / to show / to conclude that …' Do not accept 'No evidence…'. Do not accept statements such as 'there is sufficient evidence to suggest…'.
		8	No cells combined can earn a maximum of 5/8 (B1 M1 A1 M0 M1 A0 M1 A0).
2(b)	Company *A* (appears less reliable).	**B1**	
	The observed frequencies are higher than the (combined) expected frequencies for company *A* for late and cancelled, but not for company *B* and company *C*.	**DB1**	Comparison of observed and expected frequencies.
		2	

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Question	Answer	Marks	Guidance
3	$\Sigma(x-\bar{x})^2 = 29144 - \frac{1}{7}\left(445^2\right)\left[=854.7\right]$ $\Sigma(y-\bar{y})^2 = 11622 - \frac{1}{9}\left(310^2\right)\left[=944.2\right]$	**M1**	Either expression, may be embedded. May be implied by $\frac{5983}{42}$ or $\frac{4249}{36}$ or 142.45 or 118.03 seen.
	$s_p^2 = \frac{'854.7' + '944.2'}{7+9-2}$	**M1**	Correct form for pooled variance.
	128.495	**A1**	$\frac{113333}{882}$, SOI, may be implied by $s = 11.336$.
	$\left(\frac{445}{7} - \frac{310}{9}\right) \pm 2.145\sqrt{'128.5'\left(\frac{1}{7}+\frac{1}{9}\right)}$	**B1**	B1 for 2.145 seen.
		M1	Correct form, must be a *t*-value. FT *their s*. If variances combined, must use $\left(\frac{445}{7} - \frac{310}{9}\right) \pm 2.145\sqrt{\frac{5983}{42} + \frac{4249}{36}}$.
	$[16.9,\ 41.4]$	**A1**	Accept with inequality signs or open brackets or $[-41.4, -16.9]$ from $\bar{Y} - \bar{X}$. Condone $[41.4, 16.9]$. Do not accept 29.15 ± 12.25.
		6	

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Question	Answer	Marks	Guidance
4(a)	$\mathrm{F}(3)=\frac{1}{12}\times3^2-\frac{1}{108}\times3^3=0.5$ [Since $\mathrm{F}(3)=0.5$, the median of X is 3.]	**B1**	AG B0 if obtained using $a=-\frac{17}{32}$. May solve $-\frac{1}{108}x^3+\frac{1}{12}x^2=\frac{1}{2}$ but must reject $3\pm3\sqrt{3}$ as they lie outside [0, 3].
		1	
4(b)	$-\frac{1}{32}\times3^2+\frac{7}{16}\times3+a=0.5,\ a=-\frac{17}{32}$	**B1**	AG, convincingly obtained.
	$\mathrm{F}(b)=1\Rightarrow-\frac{1}{32}b^2+\frac{7}{16}b-\frac{17}{32}=1$ OR $\mathrm{F}(b)-\mathrm{F}(3)=\frac{1}{2}\Rightarrow\left[-\frac{1}{32}x^2+\frac{7}{16}x(+a)\right]_3^b=0.5\Rightarrow-\frac{1}{32}b^2+\frac{7}{16}b-\frac{33}{32}=\frac{1}{2}$	**M1**	Use $\mathrm{F}(b)=1$ or $\mathrm{F}(b)-\mathrm{F}(3)=0.5$ to form quadratic in b. May see $b^2-14b+49=0$ OE.
	$b=7$	**A1**	
		3	
4(c)	$\mathrm{f}(x)=\begin{cases}-\frac{1}{36}x^2+\frac{1}{6}x & 0\leqslant x\leqslant3,\\ -\frac{1}{16}x+\frac{7}{16} & 3<x\leqslant7,\\ 0 & \text{otherwise.}\end{cases}$	**B1**	Both parts differentiated correctly, ignore domain.
		B1FT	FT *their b*. Fully defined including correct domain, may be in terms of b. Domain must cover all reals, may overlap at boundaries.
		2	

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Question	Answer	Marks	Guidance
4(d)	$\int_0^3 \frac{1}{x}\left(-\frac{1}{36}x^2+\frac{1}{6}x\right)\mathrm{d}x+\int_3^7 \frac{1}{x}\left(-\frac{1}{16}x+\frac{7}{16}\right)\mathrm{d}x$	**B1FT**	Correct expression, FT *their* $\mathrm{f}(x)$ and limits, may be in terms of *b*.
	$\left[\frac{1}{6}x-\frac{1}{72}x^2\right]_0^3+\left[-\frac{1}{16}x+\frac{7}{16}\ln x\right]_3^7$	**M1**	Attempt to integrate both parts of *their* $\mathrm{f}(x)\times\frac{1}{x}$. Must include natural logarithm term and limits stated (may use *b*). Implied by AWRT 0.496.
	$\frac{1}{8}+\frac{7}{16}\ln\left(\frac{7}{3}\right)\left[=0.496\right]$	**A1**	AWRT 0.496
		3	

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Question	Answer	Marks	Guidance
5(a)	H_0: $\mu_d = 5$, H_1: $\mu_d > 5$ OR H_0: $\mu_A - \mu_B = 5$, H_1: $\mu_A - \mu_B > 5$	**B1**	If μ not used, must have **population** means. Allow use of μ in place of μ_d.
	d: 6, 24, 12, 8, 5, 26, 13, 2	**M1**	Calculates differences, at most 2 errors.
	$s_d^2 = \frac{1}{7}\left('1694' - \frac{'96'^2}{8}\right)\left[= \frac{542}{7} = 77.43\right]$	**M1**	Must be unbiased. $s = 8.799$
	$\left[t =\right] \frac{\frac{'96'}{8} - 5}{\sqrt{\frac{'77.43'}{8}}}$	**M1**	M0 if 5 omitted.
	2.25	**A1**	AWRT 2.25
	'2.25' > 1.415, reject H_0 / significant.	**M1**	Compare *their* 2.25 with 1.415 and appropriate conclusion (may be in terms of H_1). *Their* 2.25 must come from an attempt at standardising. Test result can be implied by an attempt at an appropriate conclusion in context ignoring hypotheses. For 2-sample *t*-test allow comparison with 1.345.
	Sufficient evidence to support Margaret's claim. OR Sufficient evidence to suggest that the course will improve a student's score by **more than 5** marks (on average).	**A1**	Correct conclusion from correct working, ignoring hypotheses. In context. Level of uncertainty in language is used (for example, not 'prove'). Do not accept statements such as "there is insufficient evidence to suggest…".
		7	Biased variance: max 4/7 (B1M1M0M1A0M1A0) 2-sample: max 2/7 (B1M0M0M0A0M1A0)

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Question	Answer	Marks	Guidance
5(b)	**Population** of **differences** are **normally** distributed.	**B1**	B1 for underlying/parent distribution of differences is normal. B0 for population **mean** is normally distributed. B0 for (underlying) **data** is normal.
		1	

Question	Answer	Marks	Guidance
6(a)	$k(a+1)^2 = 1$	***B1**	$ka^2 + 2ak + k = 1$
	$ka^2 - k = \frac{1}{3}$	***M1**	Uses $G'(1) = \frac{1}{3}$ to form equation in k and a. $-k(a+1)^2 + 2ka(a+1) = \frac{1}{3}$
	$\frac{a^2-1}{(a+1)^2} = \frac{1}{3}$	**DM1**	Uses *their* equations to eliminate k or a.
	$a = 2$	**A1**	AG, convincingly obtained.
	$k = \frac{1}{9}$	**B1**	
		5	
6(b)	**EITHER:** $G''_X(t) = \frac{2}{9t^3}$, $G''_X(1) = \frac{2}{9}\left[= 2k\right]$	**(B1FT**	CWO with *their* k. Differentiates *their* $G'_X(t)$ and evaluates at $t = 1$. May be in terms of k.
	$\text{Var}(X) = \frac{2}{9} + \frac{1}{3} - \left(\frac{1}{3}\right)^2 = \frac{4}{9}$	**B1FT)**	Uses *their* values correctly in correct formula to obtain positive value.

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Question	Answer	Marks	Guidance
6(b)	**OR:** $\mathrm{E}\left(X^2\right)=\frac{5}{9}\left[=5k\right]$	**(B1FT**	x | -1 | 0 | 1 $\mathrm{P}(X=x)$ | k | $4k$ | $4k$
	$\mathrm{Var}(X)=\frac{5}{9}-\left(\frac{1}{3}\right)^2=\frac{4}{9}$	**B1FT)**	Uses *their* values correctly in correct formula to obtain positive value.
	Available marks	**2**	
6(c)	**EITHER:** $\mathrm{G}_Y(t)=\frac{1}{729t^3}(2t+1)^6\left[=\frac{k^3}{t^3}(2t+1)^6\right]$	**(B1FT**	SOI FT *their k*. May be in terms of *k*.
	Coefficient of t^0 term: $\frac{1}{729}\binom{6}{3}(2)^3\left[=k^3\binom{6}{3}(2)^3\right]$	**M1**	Identifies appropriate term and attempt to calculate. May be in terms of *k*.
	$\mathrm{P}(Y=0)=\frac{160}{729}$	**A1)**	AWRT 0.219
	OR: Relevant values for (X_1,X_2,X_3) are (–1, 0, 1), 6 possible orderings, and (0,0,0).	**(B1**	All seven cases considered, SOI.
	$6\times\frac{1}{9}\times\frac{4}{9}\times\frac{4}{9}+\left(\frac{4}{9}\right)^3\left[=6\times k\times(4k)^2+(4k)^3\right]$	**M1**	Must be correct form using their *k*, may be in terms of *k*.
	$\frac{160}{729}$	**A1)**	
	Available marks	**3**	

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