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CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7918380717

FURTHER MATHEMATICS **9231/12**

Paper 1 Further Pure Mathematics 1 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(4r-3)(12r+3) = an^3 + bn^2 + cn\,,$$

where a, b and c are integers to be determined. [3]

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(b) Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(4r-3)(12r+3)}$ in terms of n. [4]

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(c) State the value of $\sum_{r=1}^{\infty} \frac{1}{(4r-3)(12r+3)}$. [1]

2 Prove by mathematical induction that $3^{6n} + 92^n + 89$ is divisible by 91 for all positive integers n. [6]

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3 The matrix **M** represents the sequence of two transformations in the x-y plane given by a stretch parallel to the x-axis, scale factor k ($k \neq 0, 1$), followed by a shear, x-axis fixed, with (0,1) mapped to (2,1).

(a) Show that $\mathbf{M} = \begin{pmatrix} k & 2 \\ 0 & 1 \end{pmatrix}$. [4]

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(b) The transformation represented by **M** has a line of invariant points.

Find, in terms of k, the equation of this line. [3]

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The square S in the x-y plane is transformed by **M** onto the parallelogram P.

(c) Given that the area of S is half of the area of P, find the possible values of k. [2]

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4 The cubic equation $5x^3 - 2x + 1 = 0$ has roots α, β, γ.

(a) Find a cubic equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$. [3]

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(b) Find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4} + \frac{1}{\gamma^4}$. [3]

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(c) Find also the values of $\frac{1}{\alpha^6}+\frac{1}{\beta^6}+\frac{1}{\gamma^6}$ and $\frac{1}{\alpha^8}+\frac{1}{\beta^8}+\frac{1}{\gamma^8}$. [4]

5 The lines l_1 and l_2 have equations $\mathbf{r} = -2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = -3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \mu(2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$ respectively.

(a) Find the shortest distance between l_1 and l_2. [5]

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(b) The plane Π_1 contains l_1 and the point $P(1, -1, 1)$.

The plane Π_2 contains l_2 and passes through the origin.

Find the acute angle between Π_1 and Π_2. [6]

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6 The curve C has equation $y = \dfrac{x^2 - x + 2}{4x + 4}$.

(a) Find the equations of the asymptotes of C. [3]

(b) Find the coordinates of any stationary points on C. [3]

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(c) Sketch C. [3]

(d) Sketch the curve with equation $y = \left| \frac{x^2 - x + 2}{4x + 4} \right|$. [1]

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(e) Find in exact form the set of values of x for which $\left|\frac{x^2-x+2}{4x+4}\right| < 2$. [4]

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7 The curve C has polar equation $r^2 = \dfrac{a}{1+\tan 2\theta}$ for $0 \leqslant \theta < \frac{1}{4}\pi$, where a is a positive constant.

(a) Show that, at the point on C furthest from the initial line,

$$\cos\theta - \frac{\sin\theta}{\cos 2\theta(\sin 2\theta + \cos 2\theta)} = 0$$

and verify that this equation has a root between 0.5 and 0.6. [6]

(b) Sketch C. [2]

(c) Use the substitution $v = \tan 2\theta$ to show that the area enclosed by C and the initial line is

$$\tfrac{1}{4}a\int_0^{\infty}\frac{1}{(1+v)(1+v^2)}\,\mathrm{d}v$$

and find the exact value of this area. [9]

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