* 0000800000001 * DFD

CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 9 9 0 5 2 1 7 3 4 0 *

FURTHER MATHEMATICS **9231/13**

Paper 1 Further Pure Mathematics 1 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (CJ) 355932/2
© Cambridge University Press & Assessment 2026

1 The cubic equation $x^3 + bx^2 + cx + d = 0$, where b, c and d are constants, has roots α, β and γ . It is given that

$$\begin{aligned} \alpha \quad + \beta \quad + \gamma \quad &= 5, \\ \alpha^2 \quad + \beta^2 \quad + \gamma^2 \quad &= -19, \\ \alpha^{-1} + \beta^{-1} + \gamma^{-1} &= 11. \end{aligned}$$

(a) Find the values of b, c and d. [5]

DO NOT WRITE IN THIS MARGIN

(b) Find the value of $\alpha^3 + \beta^3 + \gamma^3$. [2]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

2 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

(a) The matrix **M** represents a sequence of two geometrical transformations in the x-y plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN DO NOT WRITE IN THIS MARGIN DO NOT WRITE IN THIS MARGIN DO NOT WRITE IN THIS MARGIN DO NOT WRITE IN THIS MARGIN

(b) Find the equations of the invariant lines, through the origin, of the transformation represented by **M**. [5]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

3 The matrices **A**, **P** and **D** are given by

$$\mathbf{A} = \begin{pmatrix} 8 & -5 \\ 7 & -4 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 5 & 3 \\ 7 & 3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}.$$

Prove by mathematical induction that, for all positive integers n, $\mathbf{A}^n = \mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$. [7]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

4 **(a)** By considering $(2r+1)^4-(2r-1)^4$, use the method of differences to prove that

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2.$$ [6]

DO NOT WRITE IN THIS MARGIN

© Cambridge University Press & Assessment 2026

Let $S_n = 1^3 - 2^3 + 3^3 - 4^3 + \ldots + (-1)^{n+1} n^3$.

(b) Show that $S_{2n} = n^2(an + b)$, where a and b are integers to be determined. [3]

DO NOT WRITE IN THIS MARGIN

5 The curve C has polar equation $r^2 = \theta \sec^2 \theta$, where $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

.....

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{3}\pi$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

(c) Show that C has Cartesian equation $y = x\tan(x^2)$, where $0 \leqslant x \leqslant \frac{1}{3}\sqrt{3\pi}$. [3]

(d) Using your answer to **5(b)**, deduce the exact value of $\int_0^{\frac{1}{3}\sqrt{3\pi}} x\tan(x^2)\,\mathrm{d}x$. [3]

DO NOT WRITE IN THIS MARGIN

6 The curve C has equation $y = \dfrac{x+1}{x^2+x+1}$.

(a) Show that C has no vertical asymptotes and state the equation of the horizontal asymptote. [2]

(b) Find the coordinates of any stationary points on C. [3]

DO NOT WRITE IN THIS MARGIN

(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

.....

(d) Sketch the curve with equation $y = \left|\frac{x+1}{x^2+x+1}\right|$. [2]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

(e) Find in exact form the set of values of x for which $\left|\frac{x+1}{x^2+x+1}\right| < \frac{1}{3}$. [4]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

7 The plane Π has equation $\mathbf{r} = \mathbf{j} + 2\mathbf{k} + s(\mathbf{i} - \mathbf{j}) + t(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(a) Obtain an equation of Π in the form $ax + by + cz = d$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) The point P has position vector $(6 + 6\theta)\mathbf{i} + (1 + \theta)\mathbf{j} + (2 + 5\theta)\mathbf{k}$.

Find the set of values of θ for which the perpendicular distance of P from Π is less than $\sqrt{6}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find an equation for the common perpendicular to the lines $\mathbf{r} = \lambda(\mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \mu(-3\mathbf{i} + \mathbf{j} - \mathbf{k})$. [8]

DO NOT WRITE IN THIS MARGIN

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN



BLANK PAGE

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

© Cambridge University Press & Assessment 2026





BLANK PAGE

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

© Cambridge University Press & Assessment 2026



BLANK PAGE

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (Cambridge University Press & Assessment) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in our Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge International Education is the name of our awarding body and a part of Cambridge University Press & Assessment, which is a department of the University of Cambridge.

© Cambridge University Press & Assessment 2026