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FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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1 Find the first three terms in the Maclaurin's series for $\sinh^{-1}(x+1)$ in the form $\ln a + bx + cx^2$, giving the exact values of the constants a, b and c. [5]

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2 Let $I_n = \int_0^1 (1+t)^n \sin \pi t \, \mathrm{d}t$, where n is a non-negative integer.

(a) Show that, for $n \geqslant 2$, $\pi^2 I_n = \pi(2^n + 1) - n(n-1) I_{n-2}$. [4]

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(b) Find the exact value of I_2. [3]

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3 **(a)** Using the substitution $u = \cosh x$, find $\int \sinh^3 x \, \mathrm{d}x$. [4]

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(b) Find the particular solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\coth x = \sinh^2 x,$$

given that $y = 0$ when $x = \ln\frac{1}{3}$. Give your answer in the form $y = \mathrm{f}(x)$. [7]

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4 The curve C has equation

$$(x-y)^5 + \ln(xy) = 1 + \ln 2.$$

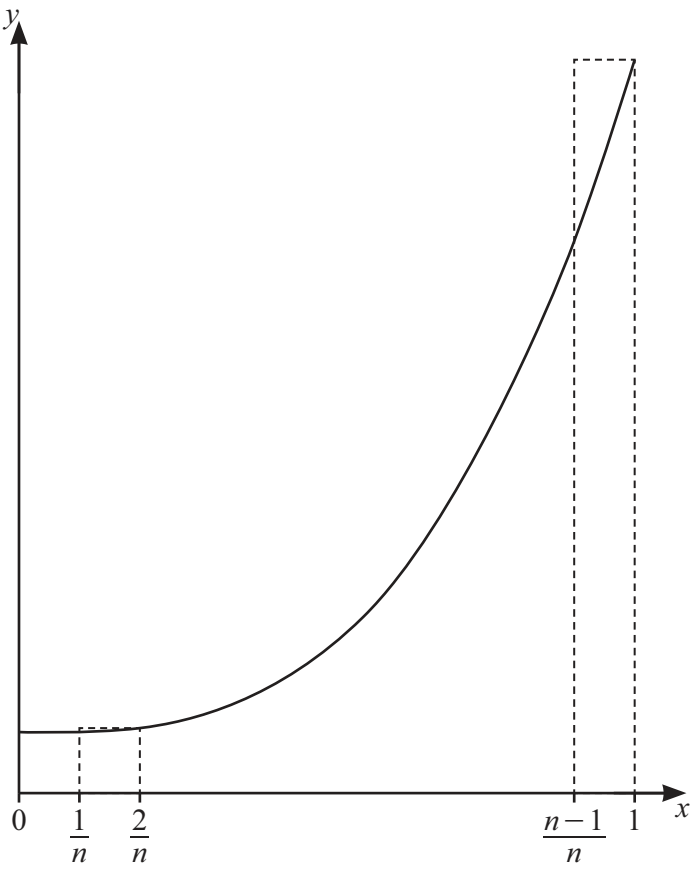
(a) Show that, at the point $(-1, -2)$ on C, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{8}{11}$. [3]

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(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$. [5]

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The diagram shows the curve with equation $y = \frac{11}{10}x^3 + \frac{1}{10}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10}\right)\mathrm{d}x < U_n$, where

$$U_n = \frac{11}{40}\left(1 + \frac{1}{n}\right)^2 + \frac{1}{10}.$$ [4]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10}\right)\mathrm{d}x$. [4]

(c) Show that $\lim_{n\to\infty}\left(U_n - L_n\right) = 0$. [2]

6 Find the particular solution of the differential equation

$$5\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4\frac{\mathrm{d}y}{\mathrm{d}x}+y=x^2+7x+1,$$

given that, when $x=0$, $y=0$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. [10]

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7 **(a)** State the sum of the series $z+z^2+z^3+...+z^n$, for $z \neq 1$. [1]

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(b) Find all roots of the equation $1+z+z^2+...+z^6=0$ in the form $e^{iq\pi}$, where q is rational. [2]

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(c) Given instead that $z = e^{-1+i\theta}$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} e^{-m}\cos m\theta = \frac{e\cos\theta - 1}{e^2 - 2e\cos\theta + 1}.$$ [7]

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8 **(a)** The constants a and b are such that the system of equations

$$\begin{aligned} -x-2y\ \ +z &= 1, \\ ax+\ \ y\ \ -z &= 2, \\ by+2z &= 3 \end{aligned}$$

does not have a unique solution. Find a in terms of b. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(b) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are integers to be determined. [4]

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(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^7 = \mathbf{PDP}^{-1}$. [7]

Additional page

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