* 0000800000001 *

CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0298623366

FURTHER MATHEMATICS **9231/22**

Paper 2 Further Pure Mathematics 2 **May/June 2026**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (DE/SG) 355943/1
© Cambridge University Press & Assessment 2026

[Turn over

1 Find the first three terms in the Maclaurin's series for $\sinh^{-1}(x+1)$ in the form $\ln a + bx + cx^2$, giving the exact values of the constants a, b and c. [5]

DO NOT WRITE IN THIS MARGIN

2 Let $I_n = \int_0^1 (1+t)^n \sin \pi t \, \mathrm{d}t$, where n is a non-negative integer.

(a) Show that, for $n \geqslant 2$, $\pi^2 I_n = \pi(2^n + 1) - n(n-1) I_{n-2}$. [4]

(b) Find the exact value of I_2. [3]

DO NOT WRITE IN THIS MARGIN

3 **(a)** Using the substitution $u = \cosh x$, find $\int \sinh^3 x \, \mathrm{d}x$. [4]

DO NOT WRITE IN THIS MARGIN

(b) Find the particular solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + y\coth x = \sinh^2 x,$$

given that $y = 0$ when $x = \ln\frac{1}{3}$. Give your answer in the form $y = \mathrm{f}(x)$. [7]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

4 The curve C has equation

$$(x-y)^5 + \ln(xy) = 1 + \ln 2.$$

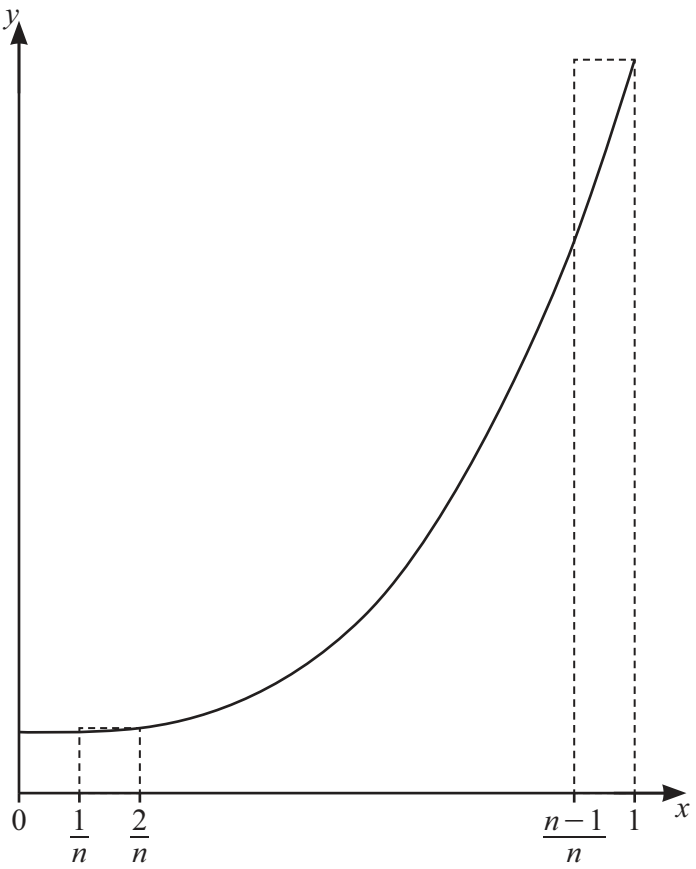
(a) Show that, at the point $(-1, -2)$ on C, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{8}{11}$. [3]

DO NOT WRITE IN THIS MARGIN

(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$. [5]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

5



The diagram shows the curve with equation $y = \frac{11}{10}x^3 + \frac{1}{10}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10}\right)\mathrm{d}x < U_n$, where

$$U_n = \frac{11}{40}\left(1 + \frac{1}{n}\right)^2 + \frac{1}{10}.$$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© Cambridge University Press & Assessment 2026

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

.....

.....

.....

.....

(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10}\right)\mathrm{d}x$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Show that $\lim_{n\to\infty}\left(U_n - L_n\right) = 0$. [2]

.....

.....

.....

.....

.....

6 Find the particular solution of the differential equation

$$5\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4\frac{\mathrm{d}y}{\mathrm{d}x}+y=x^2+7x+1,$$

given that, when $x=0$, $y=0$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. [10]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

7 **(a)** State the sum of the series $z+z^2+z^3+...+z^n$, for $z \neq 1$. [1]

.....

.....

.....

(b) Find all roots of the equation $1+z+z^2+...+z^6=0$ in the form $e^{iq\pi}$, where q is rational. [2]

.....

.....

.....

.....

.....

.....

.....

(c) Given instead that $z = e^{-1+i\theta}$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} e^{-m}\cos m\theta = \frac{e\cos\theta - 1}{e^2 - 2e\cos\theta + 1}.$$ [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

© Cambridge University Press & Assessment 2026

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

8 **(a)** The constants a and b are such that the system of equations

$$\begin{aligned} -x-2y\ \ +z &= 1, \\ ax+\ \ y\ \ -z &= 2, \\ by+2z &= 3 \end{aligned}$$

does not have a unique solution. Find a in terms of b. [3]

.....

.....

.....

.....

.....

.....

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(b) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are integers to be determined. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© Cambridge University Press & Assessment 2026

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^7 = \mathbf{PDP}^{-1}$. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (Cambridge University Press & Assessment) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in our Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge International Education is the name of our awarding body and a part of Cambridge University Press & Assessment, which is a department of the University of Cambridge.

© Cambridge University Press & Assessment 2026