CAMBRIDGE
International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2356789298

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **May/June 2026**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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1 A random sample of 9 observations of a normally distributed random variable X is as follows.

19.4 21.3 24.0 20.7 18.9 21.2 22.5 23.4 20.8

The population mean of X is μ.

Test, at the 2.5% significance level, the null hypothesis $\mu = 20$ against the alternative hypothesis $\mu > 20$. [6]

2 A large school sets an English exam for all students in a particular year group. Two teachers will be marking the scripts. To check for consistency of marking between the teachers, 10 scripts are selected at random and independently marked by each teacher. The marks awarded are shown in the following table.

student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
teacher 1	42	73	53	21	61	83	63	37	46	55
teacher 2	46	81	68	24	60	81	68	30	57	61

(a) Give a reason why a Wilcoxon matched-pairs signed-rank test might be more appropriate than a paired sample *t*-test in this situation. [1]

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(b) Carry out a Wilcoxon matched-pairs signed-rank test at the 5% significance level to investigate whether, on average, there is a difference between the marks awarded by the two teachers. [7]

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3 The discrete random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = 0.1 + 0.3t + 0.4t^2 + 0.2t^3.$$

(a) Use $G_X(t)$ to find $E(X)$ and $\text{Var}(X)$. [5]

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(b) The random variable Y is defined by $Y = 3X + 2$.

Find the probability generating function of Y. [2]

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4 A random sample of 200 observations of the continuous random variable X was taken. The values are summarised in the following table.

interval	$1 \leqslant x < 1.5$	$1.5 \leqslant x < 2$	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 3$	$3 \leqslant x < 3.5$	$3.5 \leqslant x < 4$	$x \geqslant 4$
observed frequency	97	35	23	15	10	6	14

It is required to test the goodness of fit of the distribution with probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{2}{x^3} & x \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Most of the relevant expected frequencies, correct to 2 decimal places, are given in the following table.

interval	$1 \leqslant x < 1.5$	$1.5 \leqslant x < 2$	$2 \leqslant x < 2.5$	$2.5 \leqslant x < 3$	$3 \leqslant x < 3.5$	$3.5 \leqslant x < 4$	$x \geqslant 4$
expected frequency	111.11	a	18.00	9.78	b	3.83	12.50

(a) Show that $a = 38.89$ and find the value of b correct to 2 decimal places. [3]

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(b) Carry out a goodness of fit test, at the 10% significance level, to test whether f is a satisfactory model for the data. [6]

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5 Rhiannon is investigating the mass of individual bananas sold in two supermarkets. Rhiannon claims that the mean mass of bananas sold by supermarket X is less than the mean mass of bananas sold by supermarket Y.

She takes random samples of 44 bananas from supermarket X and 52 bananas from supermarket Y. She records their masses, x kg and y kg, respectively. Her results are summarised as follows.

$$\Sigma x = 6.63 \quad \Sigma x^2 = 1.21 \quad \Sigma y = 8.56 \quad \Sigma y^2 = 1.65$$

You should not assume that the population variances of the masses of bananas sold by each supermarket are equal.

(a) Test Rhiannon's claim at the 10% significance level. [8]

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(b) Find a 90% confidence interval for the difference between the population mean masses of bananas in supermarkets X and Y. [3]

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6 The random variable X has cumulative distribution function F given by

$$\mathrm{F}(x) = \begin{cases} 0 & x < -2, \\ \frac{1}{12}(x^2 + 4x + 4) & -2 \leqslant x < 0, \\ \frac{1}{15}(x^3 + x + 5) & 0 \leqslant x \leqslant 2, \\ 1 & x > 2. \end{cases}$$

(a) Find the exact value of the lower quartile of X. [2]

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(b) Find $\mathrm{Var}\left(X^2\right)$. [5]

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(c) Find $\mathrm{P}\left(X^2 < 1\right)$. [2]

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