

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education  
Advanced Level

**FURTHER MATHEMATICS**

**9231/02**

Paper 2

October/November 2005

**3 hours**

Additional materials: Answer Booklet/Paper  
Graph paper  
List of Formulae (MF10)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your Centre number, candidate number and name on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

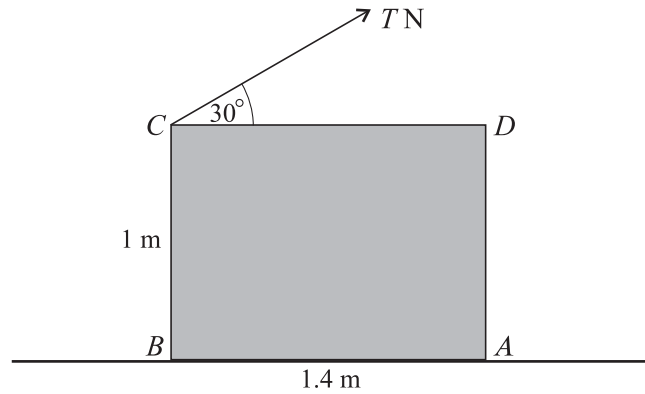
Answer **all** the questions.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
Where a numerical value is necessary, take the acceleration due to gravity to be  $10 \text{ m s}^{-2}$ .  
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.  
The use of a calculator is expected, where appropriate.  
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.  
You are reminded of the need for clear presentation in your answers.

This document consists of **6** printed pages and **2** blank pages.



1

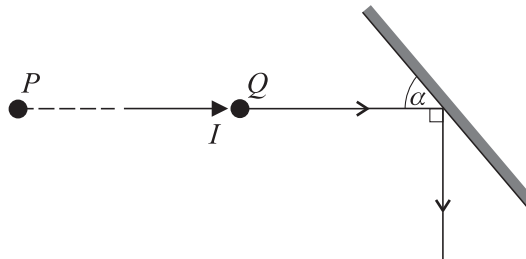


A rectangular uniform crate  $ABCD$  of mass  $1000\text{ kg}$  stands on rough horizontal ground. It is being pulled by a light chain attached to  $C$  and making an angle of  $30^\circ$  with  $CD$ . It is given that  $AB = 1.4\text{ m}$  and  $BC = 1\text{ m}$  (see diagram). The crate does not slip, and  $B$  is on the point of lifting off the ground.

(i) Find the tension  $T\text{ N}$  in the chain. [3]

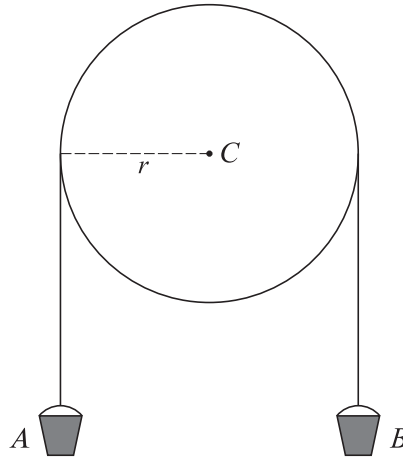
(ii) Find also the least possible coefficient of friction between the crate and the ground. [3]

- 2 Two uniform balls  $P$  and  $Q$ , of the same size but of different masses  $0.5\text{ kg}$  and  $0.3\text{ kg}$  respectively, are at rest on a smooth horizontal table.  $P$  is projected towards  $Q$  with velocity  $4\text{ m s}^{-1}$  and strikes it directly with an impulse of magnitude  $I\text{ N s}$ . The coefficient of restitution between  $P$  and  $Q$  is  $0.9$ . Find  $I$ . [4]



After the impact the ball  $Q$  continues in a straight line and strikes a smooth vertical barrier at an angle  $\alpha$  (see diagram). The coefficient of restitution between  $Q$  and the barrier is  $0.7$ . Given that the direction of motion of  $Q$  is turned through  $90^\circ$  in the collision with the barrier, find the angle  $\alpha$ . [4]

3



A uniform wheel, in the shape of a circular disc of radius  $r$  m, has mass  $12m$  kg. It is free to turn in a vertical plane about a smooth fixed horizontal axis through its centre  $C$ . A light inextensible rope passes over the upper half of the wheel. Buckets  $A$  and  $B$ , each of mass  $m$  kg, hang freely at the ends of the rope (see diagram). When the wheel is at rest, a weight of mass  $\lambda m$  kg is placed gently in  $B$ . Find the acceleration with which  $B$  begins to move, given that the rope does not slip. [6]

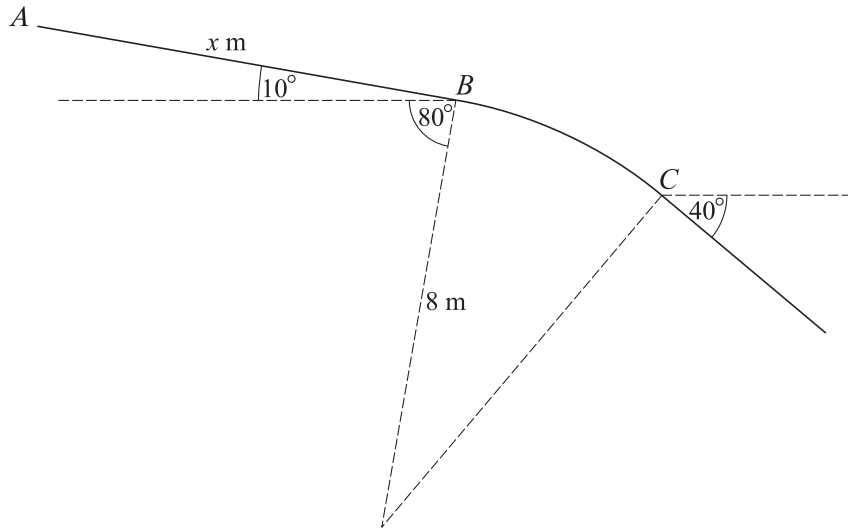
To prevent the rope slipping, the tensions on the two sides of the wheel must not differ by more than  $mg$  N. Find the greatest possible value of  $\lambda$ . [3]

- 4 A point on a screen is moving in a horizontal straight line with simple harmonic motion. The point is off the screen for some of its motion, and the centre of the motion is not clear. The speeds at which the point crosses certain vertical lines, at different distances from the left-hand edge of the screen, are measured. The results are given in the table.

|                                       |    |    |   |
|---------------------------------------|----|----|---|
| Distance from edge of screen (cm)     | 4  | 6  | 8 |
| Speed of point ( $\text{cm s}^{-1}$ ) | 20 | 15 | 0 |

Find the position of the centre of the simple harmonic motion, and the time it takes the point to move directly from the 4 cm line to the 6 cm line. [10]

5



A skateboard track consists of a straight section  $AB$ , inclined at  $10^\circ$  to the horizontal, followed by a curved section  $BC$ , over which the angle of slope increases from  $10^\circ$  to  $40^\circ$ . The curved section is part of the circumference of a circle of radius 8 m in a vertical plane through  $A$  (see diagram). A skateboarder starts from rest at  $A$ , where  $AB = x$  m. The skateboarder is modelled as a particle moving along the track under gravity with no resistance. Find the largest possible value of  $x$  so that the skateboarder does not lose contact with the curved section of the track before reaching  $C$ . [10]

- 6 A randomly chosen sample of 80 motor-cyclists were asked to keep records of the number of punctures they had in a 5-year period, together with a statement whether they kept their tyre pressures high, medium or low during that period. The table summarises their records.

|          |        | Number of punctures |             |             |
|----------|--------|---------------------|-------------|-------------|
|          |        | Less than 3         | From 3 to 5 | More than 5 |
| Pressure | Low    | 6                   | 8           | 13          |
|          | Medium | 6                   | 9           | 10          |
|          | High   | 12                  | 9           | 7           |

Test whether number of punctures is independent of tyre pressure at the 10% significance level. [6]

- 7 A researcher into diabetes plans to find a confidence interval for the mean blood pressure of all patients with diabetes. It is known from experience that the distribution of these blood pressures is normal. He examines a random sample of 8 patients and finds that their blood pressures,  $x$  mm of mercury, are summarised by  $\Sigma x = 1044$ ,  $\Sigma x^2 = 137\,336$ . Obtain a 98% confidence interval for the mean blood pressure of all patients with diabetes. [5]

The researcher wishes to reduce the width of this interval to 5 mm of mercury. Assuming that he now needs a large sample, estimate how large his random sample should be to achieve this. [3]

- 8 A horse seller claims that horses of breed  $A$  are, on average, more than 20 cm taller at the shoulder than horses of breed  $B$ . He takes a random sample from each breed and produces the figures in the table, in which  $n$  is the number of horses in the sample,  $h$  cm is the mean height at the shoulder, and  $s^2$  cm<sup>2</sup> is the unbiased estimate of the population variance.

|           | $n$ | $h$ | $s^2$ |
|-----------|-----|-----|-------|
| Breed $A$ | 6   | 165 | 104.3 |
| Breed $B$ | 4   | 136 | 55.5  |

State what you need to assume, in addition to the normality of the two populations, before applying a test at the 5% level of significance of the horse seller's claim. Carry out the test. [8]

- 9 A teacher gives two tests,  $X$  and  $Y$ , to her class of  $n$  pupils, and labels their marks  $(x_1, y_1)$ ,  $(x_2, y_2)$ , ...,  $(x_n, y_n)$  respectively. An extra pupil is added to her class and the teacher gives her the provisional marks  $(\bar{x}, \bar{y})$ , the means of the marks already awarded. Show that the regression lines of  $y$  on  $x$  before and after the extra pupil's arrival are identical. [5]

|     |    |    |    |    |
|-----|----|----|----|----|
| $x$ | 24 | 38 | 41 | 57 |
| $y$ | 34 | 21 | 63 | 22 |

Given that  $n = 4$  and that the marks are as in the table, find the regression line of  $y$  on  $x$ . [2]

The extra pupil now sits test  $X$  and scores 46. Use the regression line to estimate her mark for test  $Y$ . Comment on the suitability of the method in this case, justifying your answer. [3]

- 10 During a period of repairs on a railway, the number of trains,  $N$ , leaving a city station in a randomly chosen one-hour period is modelled by a Poisson distribution with mean  $\mu$ . Given that  $P(N = 5) = 5P(N = 3)$ , find  $\mu$ . [3]

Write down the probability that no train leaves in a randomly chosen  $t$ -hour period. [1]

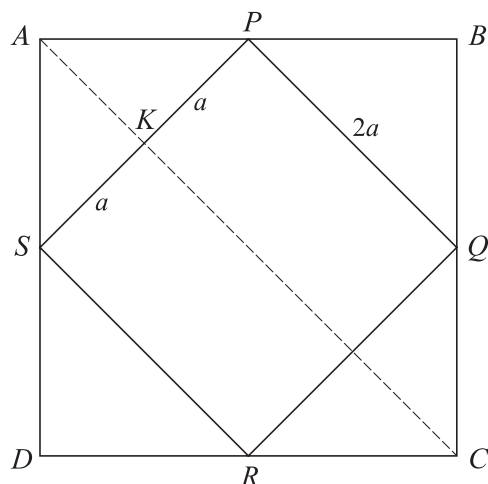
The interval between successive departures is denoted by  $T$  hours. Obtain the distribution function of  $T$ , and deduce that  $T$  has a negative exponential distribution. [2]

Given that a train has just left the station, find the probability that the next train leaves within 10 minutes. [2]

Find the median of the interval between successive departures. [3]

11 Answer only **one** of the following two alternatives.

**EITHER**



Two uniform square laminas are combined into a single body. One lamina  $ABCD$  has mass  $5m$  and the other lamina  $PQRS$  has mass  $m$ . The lamina  $PQRS$  has side  $2a$ , and its vertices are at the mid-points of the sides of  $ABCD$ , with  $P$  on  $AB$  and  $S$  on  $AD$ . The line  $PS$  meets  $AC$  at  $K$ , and the body rotates in a vertical plane about a horizontal axis  $k$  through  $K$  (see diagram). Find the moment of inertia of the body about  $k$ . [5]

(i) Assume that there is no resistance to motion. The vertex  $C$  is held vertically above  $K$  and is slightly displaced so that the body begins to rotate about  $k$ , reaching a greatest angular speed  $\omega$ . Show that  $\omega^2 = \frac{9g}{5a}$ . [3]

(ii) Assume instead that there is a resistance to motion. The vertex  $C$  is held vertically above  $K$  and is projected horizontally. When the speed of projection is  $3\sqrt{\left(\frac{ga}{5}\right)}$  the body comes to rest, after half a revolution, with  $C$  vertically below  $K$ . When the speed of projection is  $v$  the body comes to rest after making one complete revolution. The work done against the resistance when the body makes a complete revolution is twice the work done when the body makes half a revolution. Find  $v$  in terms of  $g$  and  $a$ . [6]

**OR**

When I drop 5 drawing pins at random on to a horizontal surface,  $n$  of them land point up and  $(5 - n)$  land point down. The table gives the frequencies obtained by dropping the 5 drawing pins together 100 times.

|     |   |    |    |    |    |   |
|-----|---|----|----|----|----|---|
| $n$ | 0 | 1  | 2  | 3  | 4  | 5 |
| $f$ | 1 | 10 | 18 | 41 | 21 | 9 |

Fit a binomial distribution to these figures, checking the goodness of fit at the 5% level. [9]

I now drop 100 drawing pins independently on to the same horizontal surface. Find the value of  $x$  such that the probability of  $x$  or more drawing pins landing point up is approximately 0.1. [5]

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