

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Subsidiary Level and GCE Advanced Level**

## **MARK SCHEME for the October/November 2012 series**

### **9231 FURTHER MATHEMATICS**

**9231/21**

Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

<b>Page 2</b>	<b>Mark Scheme</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE AS/A LEVEL – October/November 2012</b>	<b>9231</b>	<b>21</b>

### **Mark Scheme Notes**

Marks are of the following three types:

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol  $\nabla$  implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.  
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking  $g$  equal to 9.8 or 9.81 instead of 10.

<b>Page 3</b>	<b>Mark Scheme</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE AS/A LEVEL – October/November 2012</b>	<b>9231</b>	<b>21</b>

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

### **Penalties**

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

<b>Page 4</b>	<b>Mark Scheme</b>	<b>Syllabus</b>	<b>Paper</b>
	<b>GCE AS/A LEVEL – October/November 2012</b>	<b>9231</b>	<b>21</b>

Question Number	Mark Scheme Details	Part Mark	Total	
<b>1</b>	Find MI of disc $A$ about $O$ : Find MI of disc $B$ about $O$ : Find MI of rod $AB$ about $O$ : Find MI of body about $O$ :	$I_A = \frac{1}{2} ma^2 + m(4a)^2 [= (33/2)ma^2]$ B1 $I_B = \frac{1}{2} ma^2 + m(6a)^2 [= (73/2)ma^2]$ B1 $I_{rod} = \frac{1}{3} 3m(5a)^2 + 3ma^2 [= 28ma^2]$ B1 $I_{body} = I_A + I_B + I_{rod} = 81ma^2$ <b>A.G.</b> M1 A1	5	<b>[5]</b>
<b>2 (i)</b>	Find eqn of motion for disc: Find eqn of motion for particle: Eliminate $T$ to find angular accel.:  <b>S.R.:</b> M1 only for $1.5g \times 0.4 = 0.2 d^2\theta/dt^2$ [ $d^2\theta/dt^2 = 30$ , $(d\theta/dt)^2 = 10\pi$ , $v = 2.24$ ]	$T \times 0.4 = 0.2 d^2\theta/dt^2$ M1 $1.5g - T = 1.5 \times 0.4 d^2\theta/dt^2$ M1 $1.5g = (0.6 + 0.5) d^2\theta/dt^2$ M1 $d^2\theta/dt^2 = 15g/11$ or $13.6$ [rad s <sup>-2</sup> ] A1	4	
<b>(ii)</b>	<b>EITHER</b> Integrate to find $(d\theta/dt)^2$ : Apply initial conds. and $\theta = \pi/6$ :  <b>OR</b> Use energy to find $(d\theta/dt)^2$ :  Simplify: Find speed of particle:	$\frac{1}{2} (d\theta/dt)^2 = (15g/11)\theta [+ c]$ M1 $(d\theta/dt)^2 = 5\pi g/11$ or $14.3$ A1  $\frac{1}{2} 0.2 (d\theta/dt)^2 + \frac{1}{2} 1.5 (0.4 d\theta/dt)^2$ $= 1.5g \times 0.4 \times \pi/6$ (M1) $(d\theta/dt)^2 = 5\pi g/11$ or $14.3$ (A1) $v = 0.4 d\theta/dt = 51$ [m s <sup>-1</sup> ] B1	3	<b>[7]</b>
<b>3</b>	Use energy to find speed $v$ when $AP$ vertical: Use energy to find speed $w$ when $AP$ at angle $\theta$ :  (note that $v$ need not be found) Use $F = ma$ radially to find tension $T$ : Substitute for $w^2$ : Find $x/a$ if $T = 0$ when $\theta = \pi$ :	$\frac{1}{2}mv^2 = mga$ [ $v^2 = 2ga$ ] B1 $\frac{1}{2}mw^2 = \frac{1}{2}mv^2 - mg(a-x)(1 - \cos \theta)$ M1 A1 $[mw^2 = 2mg\{x + (a-x)\cos \theta\}]$ $T - mg \cos \theta = mw^2/(a-x)$ M1 A1 $T = mg\{3 \cos \theta + 2x/(a-x)\}$ <b>A.G.</b> M1 A1 $2x = 3(a-x)$ , $x/a = 3/5$ M1 A1	7 2	<b>[9]</b>
<b>4</b>	Resolve speeds parallel to barrier: Resolve speeds perpendicular to barrier: Find $v^2$ Relate loss of K.E. to that before collision:	$v \cos \theta = u \cos 60^\circ [= u/2]$ B1 $v \sin \theta = \frac{1}{3} u \sin 60^\circ [= u/2\sqrt{3}]$ M1 $v^2 = u^2 (1/12 + 1/4) = \frac{1}{3} u^2$ A1 $\frac{1}{2} 2m(u^2 - v^2) = \frac{2}{3} \times \frac{1}{2} 2mu^2$ <b>A.G.</b> M1 B1	5	
<b>(i)</b>	Find (reversed) speed of $P$ using impulse:	$2mw_P = \frac{2}{3}mu(1 + \sqrt{3}) - 2mv$ $w_P = \frac{1}{3}u$ <b>A.G.</b> M1 A1	2	
<b>(ii)</b>	Find (reversed) speed of $Q$ using impulse: OR by conservation of momentum:  Find coefficient of restitution:	$mw_Q = \frac{2}{3}mu(1 + \sqrt{3}) - mu$ $2mu/3 - mw_Q = -2mu/\sqrt{3} + mu$ $w_Q = (2/\sqrt{3} - 1/3)u$ (A.E.F.) M1 A1 $(w_P + w_Q) / (v + u)$ $= 2/(1 + \sqrt{3})$ or $\sqrt{3} - 1$ M1 A1	4	<b>[11]</b>

Page 5	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9231	21

Question Number	Mark Scheme Details	Part Mark	Total	
5	Find (or verify) $AP$ by equating equilibrium tensions:			
	$8mg (AP - 2a)/2a$ $= 16mg (6a - AP)/4a$ $AP = 32a/8 = 4a$	M1 A1 A1 A.G A1	3	
(i)	Apply Newton's law at general point, e.g.: (lose A1 for each incorrect term)	$m \frac{d^2x}{dt^2} = 8mg (2a - x)/2a - 16mg (2a + x)/4a$		
	Or	$m \frac{d^2y}{dt^2} = -8mg (2a + y)/2a + 16mg (2a - y)/4a$	M1 A2	
(ii)	Simplify to give standard SHM eqn, e.g.: S.R.: B1 if no derivation (max 3/6) Find period $T$ using SHM with $\omega = \sqrt{(8g/a)}$ :	$\frac{d^2x}{dt^2} = -8gx/a$ $T = 2\pi/\sqrt{(8g/a)} = \pi\sqrt{(a/2g)}$	A1 A.G M1 A1	6
	Find max speed using $\omega A$ with $A = a$ :	$v_{max} = \sqrt{(8g/a)} \times a$ $= \sqrt{(8ag)} \text{ or } 2\sqrt{(2ag)}$	M1 A1	2
			[11]	
6	(i) Find prob. that first snow falls on 20 <sup>th</sup> :	$(1 - 0.2)^{19} \times 0.2 = 0.00288$	M1 A1	2
	(ii) Find prob. that first snow falls before 5 <sup>th</sup> :	$1 - (1 - 0.2)^4 = 0.59[0]$	M1 A1	2
	(iii) Formulate condition for day $n$ of month: Take logs (any base) to give bound for $n$ : Find $n_{min}$ :	$1 - (1 - 0.2)^n \geq 0.95, 0.8^n \leq 0.05$ $n > \log 0.05 / \log 0.8$ $n > 13.4, n_{min} = 14$	M1 M1 A1	3
			[7]	
7	Integrate $f(x)$ to find $F(x)$ for $1 \leq x \leq 4$ : Relate dist. fn. $G(y)$ of $Y$ to $X$ for $1 \leq x \leq 4$ :	$F(x) = x^2/15 + c = (x^2 - 1)/15$ $G(y) = P(Y < y) = P(X^3 < y)$ $= P(X < y^{1/3}) = F(y^{1/3})$ $= (y^{2/3} - 1)/15$	M1 A1 A.G M1 A1	4
	(i) Find relation for median $m$ of $Y$ : Evaluate $m$ :	$G(m) = 1/2, m^{2/3} = 17/2$ $m = 24.8$	M1 A1 A1	3
	(ii) EITHER Find $g(y)$ and formulate $E(Y)$ : OR Formulate $E(Y)$ in terms of $X$ :	$g(y) = 2y^{-1/3}/45$ $E(Y) = \int yg(y)dy = \int 2y^{2/3}/45 dy$ $E(Y) = E(X^3) = \int 2x^4/15 dx$	M1 A1 (M1 A1)	
	Integrate and apply limits:	$E(Y) = \left[ \frac{2y^5}{75} \right]_1^{64} \text{ or } \left[ \frac{2x^5}{75} \right]_1^4$ $= 2(1024 - 1)/75$ $= 682/25 \text{ or } 27.3$	M1 A1	4
			[11]	

Page 6	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9231	21

8	(i)	Calculate gradient $b$ in $y - \bar{y} = b(x - \bar{x})$ :  Find regression line:	$b = (761.3 - 72.4 \times 78/8)/(769.9 - 72.4^2/8)$ M1 $= 55.4/114.68$ [or 6.925/14.335] $= 1385/2867 \text{ or } 0.483[1]$ A1 $y - 9.75 = 0.483(x - 9.05)$ Or $y = 5.38 + 0.483x$ M1 A1	4	
	(ii)	Find correlation coefficient $r$ :	$r = (761.3 - 72.4 \times 78/8) / \sqrt{\{(769.9 - 72.4^2/8)(820 - 78^2/8)\}}$ M1 $= 55.4 / \sqrt{(114.68 \times 59.5)}$ [or 6.925 / $\sqrt{(14.335 \times 7.4375)}$ ] A1 $= 0.671$ *A1	3	
	(iii)	State both hypotheses: State or use correct tabular one-tail $r$ value: Valid method for reaching conclusion: Correct conclusion (AEF, dep *A1, *B1):	$H_0: \rho = 0, H_1: \rho > 0$ B1 $r_{8, 5\%} = 0.621$ *B1 Reject $H_0$ if $ r  >$ tabular value M1 There is positive correlation A1	4	[11]
9		Estimate population variance using $A$ 's sample: (allow use of biased here: 1.489 or 1.22 <sup>2</sup> )	$s_A^2 = (481.1 - 57.4^2/7) / 6$ $= 521/300 \text{ or } 1.737 \text{ or } 1.318^2$ M1 A1		
		Find confidence interval:	$57.4/7 \pm t \sqrt{(s_A^2 / 7)}$ M1		
		State or use correct tabular value of $t$ :	$t_{6, 0.975} = 2.447$ [or 2.45] A1		
		Evaluate C.I. correct to 3 s.f.:	$8.2 \pm 1.22 \text{ or } [6.98, 9.42]$ A1	5	
		State suitable assumptions (A.E.F.):	Population of $B$ is Normal and has same variance as for $A$ B1		
		State hypotheses:	$H_0: \mu_A = \mu_B, H_1: \mu_A > \mu_B$ B1		
		Estimate population variance using $B$ 's sample: (allow use of biased here: 0.988 or 0.994 <sup>2</sup> )	$s_B^2 = (278.74 - 37^2/5) / 4$ $= 1.235 \text{ or } 1.111^2$ B1		
		Estimate population variance for combined sample:	$s^2 = (6s_A^2 + 4s_B^2) / 10$ $= 192/125 \text{ or } 1.536 \text{ or } 1.239^2$ M1 A1		
		Calculate value of $t$ (to 2 d.p.):	$t = (57.4/7 - 37/5) / s \sqrt{(1/7 + 1/5)}$ $= 0.8/0.726 = 1.10[2]$ *A1		
		State or use correct tabular value	$t_{10, 0.95} = 1.812$ [or 1.81] *B1		
	Correct conclusion (AEF, dep *A1, *B1):	$\mu_A \text{ is not greater than } \mu_B$ B1			
	S.R.: Deduct only A1 if intermediate result to 3 s.f.				
	S.R.: Invalid method for calculating $t$ (max 6/9):	$t = 0.8 / \sqrt{(s_A^2 / 7 + s_B^2 / 5)}$ $= 0.8/0.704 = 1.14$ (M1) (A1)	9	[14]	

Page 7	Mark Scheme	Syllabus	Paper
	GCE AS/A LEVEL – October/November 2012	9231	21

10 (a)	<p>For <math>A</math>, let contact pts with plane, sphere <math>C</math> be <math>P, S</math>            Stating or implying reactions <math>R_P, R_S</math> same as for <math>B</math>:            Stating or implying <math>F_P = F_S</math> by moments about <math>O_A</math>:            Stating or implying 3 indep. eqns for <math>F, R_P, R_S</math> e.g.:</p>	<p>B1            B1            3 × M1 A1</p>	14	[14]
	<p>Up to 2 resolutions of forces, e.g. <math>\uparrow</math> for system: <math>2R_P = 3W</math>  <math>\uparrow</math> for <math>A</math>: <math>R_P = W + R_S \cos \theta + F_S \sin \theta</math>  <math>\uparrow</math> for <math>C</math>: <math>2R_S \cos \theta + 2F_S \sin \theta = W</math>  <math>O_A \rightarrow O_C</math> for <math>A</math>: <math>R_P \cos \theta + F_P \sin \theta = R_S + W \cos \theta</math></p>			
	<p>Moments about <math>S</math> for <math>A</math>:  <math>F_P (r + r \cos \theta) + Wr \sin \theta</math>  <math>= R_P r \sin \theta</math></p>			
	<p>Find <math>R_P</math>: <math>R_P = 3W/2</math> A1</p>			
	<p>Find <math>R_S</math>: <math>R_S = W/2</math> A1</p>			
	<p>Find <math>F</math> at <math>P</math> and/or <math>S</math>: <math>F = (W \sin \theta) / 2(1 + \cos \theta)</math> A1</p>			
	<p>Use <math>F \leq \mu R_P</math> to find bound for <math>\mu</math>: <math>\mu \geq \sin \theta / 3(1 + \cos \theta)</math> A.G. M1 A1</p>			
	<p>Use <math>F \leq \mu' R_S</math> to find bound for <math>\mu'</math>: <math>\mu' \geq \sin \theta / (1 + \cos \theta)</math> A.G. M1</p>			
(b)	<p>Find <math>E(X)</math> using <math>\int xf(x)dx</math>:  <math>E(X) = \int_{-2}^3 (5x^2 - x^3 - 4x)/10 dx</math> M1 A1  <math>= \frac{1}{2}(4^3 - 2^3) - 3(4^4 - 2^4)/40 - 3(4^2 - 2^2)/5</math>  <math>= 28 - 18 - 7.2 = 2.8</math> *A1</p>		5	
	<p>Verify <math>E(X)</math> within 10% of 2.69 (A1 dep *A1):  <math>(E(X) - 2.69)/2.69 = 0.041 &lt; 0.1</math>  <math>or 1.1 \times 2.69 = 2.96 &gt; E(X)</math> M1 A1</p>			
	<p>Show derivation of tabular entry:  <math>60 \int_{3.2}^{3.6} (5x - x^2 - 4)/10 dx</math> M1  <math>= 60[3(5x^2/2 - x^3/3 - 4x)/10]</math> 3.2 3.6  <math>or [45x^2 - 6x^3 - 72x]</math> 3.2 3.6  <math>= 122.4 - 83.328 - 28.8</math>  <math>or 60 \times 0.1712</math>  <math>= 10.272</math> A.G. A1</p>		2	
	<p>State (at least) null hypothesis: <math>H_0: f(x)</math> fits data (A.E.F.) B1</p>			
	<p>Combine last 2 cells since exp. value <math>&lt; 5</math>:  <math>O: \dots 8</math>  <math>E: \dots 14.208</math> B1</p>			
	<p>Calculate <math>\chi^2</math> (to 2 d.p.):  <math>\chi^2 = 0.8126 + 0.0584 + 0.2011</math>  <math>+ 2.7135 = 3.78[47]</math> M1 *A1</p>			
	<p>State or use consistent tabular value (to 2 d.p.):  <math>\chi_{3, 0.9}^2 = 6.25[1]</math>  <math>[or if no cells combined:</math>  <math>\chi_{4, 0.9}^2 = 7.78]</math> *B1</p>			
	<p>Valid method for reaching conclusion: Accept <math>H_0</math> if <math>\chi^2 &lt;</math> tabular value M1</p>			
	<p>Conclusion (A.E.F., dep *A1, *B1): <math>3.78 &lt; 6.25</math> so <math>f(x)</math> does fit A1</p>		7	[14]