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Mark Scheme (Results)

Summer 2015

Pearson Edexcel International GCSE
Further Pure Mathematics (4PM0)

Paper 1

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
    - M marks: method marks
    - A marks: accuracy marks
    - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
    - cao – correct answer only
    - ft – follow through
    - isw – ignore subsequent working
    - SC - special case
    - oe – or equivalent (and appropriate)
    - dp – dependent
    - indep – independent
    - eeoo – each error or omission
- **No working**

  If no working is shown then correct answers normally score full marks.
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
  If there is a wrong answer indicated always check the working in the body of the script and award any marks appropriate from the mark scheme.

  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

  Any case of suspected misread loses two A (or B) marks on that part, but can gain the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

  If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

  If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**
  Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

  Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**
  Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part ofthe question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2 + bx + c) = (x + p)(x + q)$where$|pq| = |c|$

$(ax^2 + bx + c) = (mx + p)(nx + q)$where$|pq| = |c|$ and $|mn| = |a|$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to

3. Completing the square:

Solving $x^2 + bx + c = \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c$ where $q \neq 0$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1.

2. Integration:

Power of at least one term increased by 1.

Use of a formula:

Generally, the method mark is gained by:

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implicationfrom the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Scheme	Marks
1.	$4x^2 - 9 = 0 \quad x = (\pm)\frac{3}{2}$ or $\frac{3}{2}$ seen as upper limit	B1
	$V = \int_0^{\frac{3}{2}} \pi y^2 \mathrm{d}x = \pi \int_0^{\frac{3}{2}} (4x^2 - 9)^2 \mathrm{d}x$	M1
	$= \pi \int_0^{\frac{3}{2}} (16x^4 - 72x^2 + 81) \mathrm{d}x$	A1
	$= \pi \left[\frac{16}{5}x^5 - 24x^3 + 81x \right]_0^{\frac{3}{2}}$	M1d
	$= 203.57... = 204$ (units^3)	A1
		[5]

Notes

B1 for $x = \frac{3}{2}$ allow $\{ x = \pm\frac{3}{2} \}$. Award when seen anywhere in the question.

M1 for a correct statement for the volume of revolution, which **must** include π **and** the function squared. Ignore limits for this mark. Ignore a missing d*x*.
If π is seen at the end of the question, (you will see this) award this mark.

A1 for a fully correct expanded expression as shown for the volume of revolution with **both** correct limits. You may not see this expression. The mark can be awarded as implied by the **correct** integrated expression seen.

M1d for an attempt at integrating their expression for the volume, which must contain as a minimum, Ax^4 as their highest power of *x*, and π. Award for $x^n \to x^{n+1}$ seen in one term in *x*, or even for their $81 \to 81x$.

Note: this M mark is dependent on the first being awarded.

A1 204 (units^3) cao Do **NOT** accept an answer of 204 (units^3) with no integration seen. If the volume is left as negative withhold this mark. If they change a negative to a positive (due to limits being wrong way around), then you can award this mark.

Question Number	Scheme	Marks
2.(a)	$\frac{dy}{dx} = 8xe^{2x} + 8x^2e^{2x}$	M1A1A1 (3)
(b)	$x\frac{dy}{dx} = 8x^2e^{2x} + 8x^3e^{2x}$	
	$= 8x^2e^{2x}(1+x)$	M1
	$= 2y(1+x)$ *	A1cso (2)
	ALT: Reverse argument: M1 correct method; A1 fully correct	**[5]**

Notes

M1 for an attempt at product rule. There must be two terms added. There must be an attempt at differentiating BOTH terms (usual rules for differentiation)

A1 for ONE term correct, need **not** be simplified

A1 for BOTH terms correct, need **not** be simplified. Award when seen and isw any attempts at simplification.

M1 for multiplying their $\frac{dy}{dx} = 8xe^{2x} + 8x^2e^{2x}$, through by x on BOTH sides

A1 for a correct factorized expression. **Note**: this is a show question so look out for 'fudging' of their work to achieve the given answer.

ALT

You will see attempts working from the given answer which is fine.

M1 For substituting $4x^2e^{2x}$ substituted into $2y(1+x)$ to give;

$2\times 4x^2e^{2x}(1+x) \Rightarrow 8x^2e^{2x} + 8x^3e^{2x}$

A1 for multiplying their $\frac{dy}{dx}$ by x **and** comparing the result, to verify

Question Number	Scheme	Marks
3.(a)	$4x^2 - 8x + 7 = l\left(x^2 - 2mx + m^2\right) + n$	M1
	$l = 4 \quad 2ml = 8 \quad m = 1$	A1
	$lm^2 + n = 7 \quad n = 3$	A1 (3)
	ALT: $4\left(x^2 - 2x\right) + 7 = 4\left(x - 1\right)^2 + 3$	M1A1A1 (3)
(b)	(i) $\mathrm{f}\left(x\right)_{\min} = 3$	B1ft
	(ii) when $x = 1$	B1ft (2)
		[5]

Notes

(a)

Note: there is only one method mark in part (a). The method MUST be complete for award of this mark

M1 for setting the given expression or f(*x*) equal to $l\left(x - m\right)^2 + n$ and attempting to expand the $\left(x - m\right)^2$ into 3 terms, ie., $\left(x^2 \pm Amx \pm m^2\right)$ where $A \neq 0$

A1 for the values of l = 4, **and** m = 1.
Accept embedded values. If there is an error transferring the correct embedded value, isw.

A1 for the value of n = 3

ALT

M1 for taking 4 as a common factor of the term in x^2 and $2x$, **and** attempting to complete the square (usual rules – please refer to General Guidance)

A1 for achieving $4\left[\left(x - 1\right)^2 - 1\right] + 7$ Penalise poor bracketing unless final answer is correct.

A1 for the final answer { f(*x*)} = $4\left(x - 1\right)^2 + 3$. Accept answers embedded in the expression. If there is an error transferring the correct embedded value, isw.

(b)

B1ft for minimum = 3 (ft their value of n)

B1ft for x = 1 (ft their value for m)

There must be no transposition of the 3 and the 1 unless it is clear they write $\mathrm{f}(x) = 3$ and $x = 1$

Question Number	Scheme	Marks
4(a)	$a = 18$	B1 (1)
(b)	$S_2 = 2a + d = 2\times 2(10-2)$	M1
	$36 + d = 32 \quad d = -4$	A1 (2)
(c) (i)	$S_n = 2n(10-n) > -50$	M1
(ii)	$20n - 2n^2 > -50$	
	$n^2 - 10n - 25 < 0$	A1
	Crit values $n = \frac{10 \pm \sqrt{100+100}}{2} = 12.07, \ -2.07$	M1
	Greatest $n = 12$	A1 (4)
		[7]

<u>Notes</u>

(a) B1 a = 18 only

(b)

M1 for $a + (a+d) = 2\times 2(10-2)$ ft their a. The method **must** be complete.

A1 for $d = -4$ **Note: *d* = 14 is M0A0**

ALT Method of differences

M1 for correct values of S_n seen in a table **and** finding first **and** second differences.

A1 for at least three −4's seen with no incorrect values.

(c)

(i) M1 for writing down the correct inequality as shown

A1 for forming a 3TQ with the correct inequality as shown

(ii) M1 for attempting to solve the 3TQ either by formula or completing the square.

(Please refer to General Guidance for the definition of an attempt)

Note: Attempts to factorise using integers are MO

A1 for n = 12. If they also offer n = −2 then this is A0. Do not isw.

Special Case: Answer only of *n* = 12, or using trial and error and giving *n* = 12 is M1A1

Question Number	Scheme	Marks
5(a)	$(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=\alpha^3-\alpha^2\beta+\alpha\beta^2+\beta\alpha^2-\beta^2\alpha+\beta^3=\alpha^3+\beta^3$	B1 (1)
(b)	$\alpha+\beta=\frac{-6}{2}=-3\ \alpha\beta=-\frac{7}{2}$	B1
	$\alpha^3+\beta^3=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$	M1
	$=(-3)\left((-3)^2-3\times\frac{-7}{2}\right),=-\frac{117}{2}$	A1ft,A1 (4)
(c)	$(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2=(\alpha+\beta)^2-4\alpha\beta=(-3)^2-4\times\frac{-7}{2}=23$	M1
	$\alpha-\beta=\sqrt{23}$	A1 (2)
(d)	$(\alpha^3-\beta^3)=(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)=(\alpha-\beta)\left((\alpha+\beta)^2-\alpha\beta\right)$	M1
	$=\sqrt{23}\left((-3)^2+\frac{7}{2}\right)=\frac{25}{2}\sqrt{23}$	A1 (2) **[9]**
	ALT: $2\alpha^3+6\alpha^2-7\alpha=0$ and $2\beta^3+6\beta^2-7\beta=0$	
	Subtract and substitute	M1
	Correct answer	A1

Notes

NOTE: If they use the quadratic formula to answer any part of the question, award zero marks in that part.

(a) B1 for simplification as shown cso. This is a show question so multiplication, ie., $\alpha^3 - \alpha^2\beta + \alpha\beta^2 + \beta\alpha^2 - \beta^2\alpha + \beta^3$ must be seen.

(b)

B1 for both sum and product (the sum need not be simplified to −3)

M1 If they use the given answer in part (a) , they must achieve an expansion that is as a minimum $(\alpha+\beta)\left\{(\alpha+\beta)^2 + A\alpha\beta\right\}$ where $A \neq 0$

For an fresh attempt at an expansion and simplification of $(\alpha+\beta)^3$. Minimally acceptable attempt; $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - A\alpha\beta(\beta+\alpha)$ where $A \neq 0$

{Note: $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ is the correct expansion}

Note: Their attempt must have been sufficiently simplified in order to substitute their sum in terms of $(\alpha+\beta)$ and $\alpha\beta$

A1ft for substituting their values for the Sum and Product into their $\alpha^3 + \beta^3$.

A1 for the correct answer $\left\{-\frac{117}{2}\right\}$oe

(c)

M1 for expanding $(\alpha-\beta)^2 = (\alpha+\beta)^2 - B\alpha\beta$, where $B \neq 2$ or 0 **AND** for substituting **their** values for the Sum and Product **OR** for $(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$

A1 for answer as shown $\alpha - \beta = \sqrt{23}$

(d)

M1 for the **correct algebra** on the expansion of $(\alpha-\beta)^3$ to give either;

$(\alpha-\beta)\left((\alpha+\beta)^2 - \alpha\beta\right)$ or $(\alpha-\beta)^3 - 3\alpha\beta(\alpha+\beta)$ or $(\alpha-\beta)\left(\alpha^2 + \alpha\beta + \beta^2\right)$

A1 final answer as shown

ALT

M1 uses the given equation and substitutes α and β, and subtracts to give $\alpha^3 - \beta^3 = \frac{7(\alpha-\beta) - 6\left(\alpha^2 - \beta^2\right)}{2}$ and substitutes their values of $(\alpha-\beta)$ and $\left(\alpha^2 - \beta^2\right)$ leading to a value for $\alpha^3 - \beta^3$

A1 for the correct answer as shown.

Question Number	Scheme	Marks
6(a)	$\cos C = \dfrac{20^2 + 14^2 - 22^2}{2 \times 20 \times 14}$	M1
	$C = 78.46304... = 78.463°$	A1
	$\dfrac{\sin A}{20} = \dfrac{\sin 78.46}{22}$	M1
	$A = 62.9643... = 62.964°$	A1
	$B = 180 - (78.463 + 62.964) = 38.573°$	A1ft (5)
(b)	$\angle APC = 180 - (31.48 + 78.46) = 70.06$	M1
	$\dfrac{AP}{\sin 78.46} = \dfrac{14}{\sin 70.06}$	M1
	$AP = 14.59... = 14.6$	A1 (3)
(c)	$\text{Area} = \dfrac{1}{2} \times 22 \times 14 \sin 62.96$	M1
	$\text{Area} = 137.1... = 137$ cm^2	A1 (2) **[10]**
	ALT: Use Heron's formula	

Notes

(a)

M1 uses a correct COS rule formula for **any** angle of the triangle. If there are errors in substitution, the correct formula must be seen first.

A1 for one of awrt C = 78.5, or B = 38.6 , or A = 63.0

M1 uses COS rule again, or SIN rule. If SIN rule is used ft their first angle for this mark. If there are errors in substitution, the correct formula must be seen first.

A1 for one of awrt C = 78.5, or B = 38.6 , or A = 63.0

A1ft uses 180 – the two angles already found. Follow through their angles for this mark. If they use trigonometry, their angles **must** add to **exactly** 180.

Both M marks must be scored for this A mark to be awarded.

Working in radians is acceptable and correct, but angles must be to awrt 3 sf.

Angle $A = 1.099^c$ Angle $B = 0.673^c$ Angle $C = 1.369^c$

(b)

M1 for APC = 180 – (their A ÷ 2 + their B) ABP = 180 – (their A ÷ 2 + their C)

M1 for using either SIN or COS rule to find the length of AP. {**Note** $PB \neq 10$ cm}

A1 for the answer as shown correctly rounded.

(c)

M1 uses correct formula for Area of a triangle = $\frac{1}{2}ab\sin C$

A1 for answer as shown 137 (cm^2).

ALT

M1 using Heron's formula,

$$s = \frac{22+20+14}{2} = 28 \Rightarrow \text{Area} = \sqrt{28(28-22)(28-20)(28-14)}$$

A1 Area = 137.171…= 137 (cm^2)

Question Number	Scheme	Marks
7 (a)	$\left(1+\frac{x}{3}\right)^{\frac{1}{4}}=1+\frac{1}{4}\times\frac{x}{3}+\frac{\frac{1}{4}\times\left(-\frac{3}{4}\right)}{2!}\left(\frac{x}{3}\right)^2+\frac{\frac{1}{4}\times\left(-\frac{3}{4}\right)\left(-\frac{7}{4}\right)}{3!}\left(\frac{x}{3}\right)^3$	M1
	$=1+\frac{x}{12}-\frac{x^2}{96}+\frac{7}{3456}x^3$ oe for each coeff	A2 (-1 ee) (3)
(b)	$\left(1-\frac{x}{3}\right)^{-\frac{1}{4}}=1+\left(-\frac{1}{4}\right)\times\left(-\frac{x}{3}\right)+\frac{\left(-\frac{1}{4}\right)\times\left(-\frac{5}{4}\right)}{2!}\left(-\frac{x}{3}\right)^2+\frac{\left(-\frac{1}{4}\right)\times\left(-\frac{5}{4}\right)\left(-\frac{9}{4}\right)}{3!}\left(-\frac{x}{3}\right)^3$	M1
	$=1+\frac{x}{12}+\frac{5}{288}x^2+\frac{5}{1152}x^3$ oe for each coeff	A2 (-1 ee) (3)
(c)	$\lvert x\rvert<3$	B1 (1)
(d)	$\left(\frac{3+x}{3-x}\right)^{\frac{1}{4}}=\left(\frac{1+\frac{x}{3}}{1-\frac{x}{3}}\right)^{\frac{1}{4}}=\left(1+\frac{x}{3}\right)^{\frac{1}{4}}\times\left(1-\frac{x}{3}\right)^{-\frac{1}{4}}$	M1
	$=\left(1+\frac{x}{12}-\frac{x^2}{96}+\frac{7}{3456}x^3\right)\times\left(1+\frac{x}{12}+\frac{5}{288}x^2+\frac{5}{1152}x^3\right)$	
	$=1+\frac{x}{12}+\frac{5}{288}x^2+\frac{x}{12}+\frac{x^2}{144}-\frac{x^2}{96}$	M1
	$=1+\frac{x}{6}+\frac{x^2}{72}$ oe for each coeff	A1 (3)

(e)	$\int_0^{0.6}\left(\frac{3+x}{3-x}\right)\mathrm{d}x = \int_0^{0.6}\left(1+\frac{x}{6}+\frac{x^2}{72}\right)\mathrm{d}x$	M1
	$=\left[x+\frac{x^2}{12}+\frac{x^3}{216}\right]_0^{0.6}$	A1
	$=0.6+\frac{0.6^2}{12}+\frac{0.6^3}{216}-0$	M1d
	$=0.631$	A1 (4) **[14]**

Notes

(a)

M1 for using a correct binomial expansion at least up to the term in x^2. If there are errors in substitution, withhold this mark if the formula is not seen. Each term, must have at least, the correct power of $\frac{x}{3}$. The expansion must start with 1.

A1 for 1 + the term in x correct, and either term in x^2 **or** x^3 correct, need not be simplified.

A1 for the expansion correct as shown above. Accept equivalent fractions.

(b)

M1 for using a correct binomial expansion at least up to the term in x^2. If there are errors in substitution, withhold this mark if the formula is not seen. Each term, must have at least, the correct power of $\frac{x}{3}$. The expansion must start with 1.

A1 for 1 + the term in x correct, and either term in x^2 **or** x^3 correct, need not be simplified.

A1 for the expansion fully correct as shown above. Accept equivalent fractions.

(c)

B1 for $-3<x<3$ or $|x|<3$

(d)

M1 for setting $\left(\frac{3+x}{3-x}\right)^{\frac{1}{4}} = \left(1+\frac{x}{3}\right)^{\frac{1}{4}} \times \left(1-\frac{x}{3}\right)^{-\frac{1}{4}}$ or their (a) × their (b)

{only need terms as far as x^2 } If there is a 3 or a 9 present, M0

M1 for multiplying out their (a) × (b). (Ignore the presence of a 3 or a 9 for this mark) Check that they have multiplied out fully. There are six terms in total up to and including terms in x^2.

A1 for the answer as shown (oe)

(e)

M1 for using their answer in part (d) to form an integral of a quadratic expression. Ignore limits and condone a missing dx.

A1 for a correct integration (no ft)

M1d for correct substitution of 0.6. Allow missing 0

A1 0.631 cso

If 0.631 is seen without working, **no marks** in part (e). (Not 'hence obtain....')

Question Number	Scheme	Marks
8(a)(i)	$\cos 2A = \cos^2 A - \sin^2 A$	M1
	$= (1-\sin^2 A) - \sin^2 A, = 1 - 2\sin^2 A$ *	M1,A1 (3)
(ii)	$\sin 2A = 2\sin A\cos A$	B1 (1)
(b)	$\sin 3A = \sin 2A\cos A + \cos 2A\sin A$	M1
	$= \sin A(1-2\sin^2 A) + 2\sin A\cos^2 A$	M1
	$\sin A - 2\sin^3 A + 2(1-\sin^2 A)\sin A$	M1
	$= 3\sin A - 4\sin^3 A$	A1 (4)
(c)	$\sin 3A = -\frac{1}{2}$	M1
	$3A = 210°, -30°, -150°$	M1 (any one)
	$A = 70°, -10°, -50°$	A1A1 (4)
(d)	(i) $\int \sin^3\theta \, d\theta = \frac{1}{4}\int (3\sin\theta - \sin 3\theta) d\theta$	M1
	$= \frac{1}{4}\left[-3\cos\theta + \frac{1}{3}\cos 3\theta\right]$	M1A1
	(ii) $\frac{1}{4}\left[-3\cos\frac{\pi}{4} + \frac{1}{3}\cos\frac{3\pi}{4} - \left(-3\cos 0 + \frac{1}{3}\cos 0\right)\right]$	
	$\frac{1}{4}\left[-\frac{3}{\sqrt{2}} - \frac{1}{3}\times\frac{1}{\sqrt{2}} - \left(-3 + \frac{1}{3}\right)\right]$	M1
	$\frac{8-5\sqrt{2}}{12}$ oe for 5,8,12	A1 (5)

(d)	ALT for (d) (i) $\int \sin^3\theta \,\mathrm{d}\theta = \int\left(\sin\theta - \cos^2\theta\sin\theta\right)\mathrm{d}\theta$ $= \left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]$ (+ *c*) (ii) $-\cos\frac{\pi}{4} + \frac{1}{3}\cos^3\frac{\pi}{4} - \left(\cos 0 + \frac{1}{3}\cos^3 0\right)$ $= -\frac{1}{\sqrt{2}} + \frac{1}{3}\times\frac{1}{2\sqrt{2}} - \left(1 + \frac{1}{3}\right) = \frac{8-5\sqrt{2}}{12}$	M1 M1A1 M1dd A1 **[17]**

<u>**Notes**</u>

(a) (ii)

M1 for the correct expression for cos 2*A*

M1 for using $\cos^2 A + \sin^2 A = 1$, and substituting into the expression for cos 2*A*

A1 for the correct identity as shown. Note this is show question!

(ii)

B1 for the correct identity for sin 2*A*

(b)

M1 for substituting sin 2*A* into their expression in part (a) to give sin (2*A* +*A*)

M1 for using the given cos 2*A* and their sin 2*A* in their expression for sin3*A*

M1 for using $\cos^2 A + \sin^2 A = 1$

A1 for the final identity as shown. Note: this is a show question

(c)

M1 for $8\sin^3 A - 6\sin A = 1 \Rightarrow -2\left(3\sin A - 4\sin^3 A\right) = 1 \Rightarrow \sin 3A = k$, where $-1 \le k \le 1$

M1 for 3*A* equal to any one of 210º, −30º, −150º

A1 for any **two** correct angles

A1 for **all three** correct angles

If there are extra angles outside of range, ignore. If there are extra angles within the range deduct one A mark for each up to a maximum of 2 marks.

(d) (i)

M1 for re-arranging the GIVEN expression for sin 3*A* to make $\sin^3 A$ the subject

M1 for an attempt at integrating their re-arranged expression

As a minimum for this mark, $\int \sin 3A \,\mathrm{d}A \Rightarrow \pm\frac{1}{3}\cos 3A\ (+c)$ (+c not required)

A1 for the correct integrated expression as shown (+c not required)

(ii)

M1dd for substituting $\frac{\pi}{4}$ **and** 0 into their integrated expression **and** attempting to evaluate (there must be a final answer given for this mark) .

A1 for the final answer as shown

ALT

(i)

M1 for finding $\sin^3 A = \sin A\left(1-\cos^2 A\right) = \sin A - \sin A\cos^2 A$

M1 for attempting to integrate. $\int \sin A\,\mathrm{d}A - \int \cos^2 A \sin A\,\mathrm{d}A = \left\{-\cos A + \frac{1}{3}\cos^3 A(+c)\right\}$

For a minimum attempt you need to see; $\int \cos^2 A \sin A\,\mathrm{d}A = \pm\frac{1}{3}\cos^3 A\ (+c)$

A1 for $-\cos A + \frac{1}{3}\cos^3 A(+c)$

(ii)

M1dd for substituting $\frac{\pi}{4}$ **and** 0 into their integrated expression **and** attempting to evaluate (there must be a final answer given for this mark) .

A1 for the final answer as shown

ALT (Using substitution)

(i)

M1 for writing the integral as $\sin\theta\sin^2\theta \Rightarrow \sin\theta\left(1-\cos^2\theta\right)$ and substituting $\cos\theta = u$ and differentiating to achieve $\frac{\mathrm{d}u}{\mathrm{d}\theta} = -\sin\theta$

M1 for substituting and integrating $-\int \sin\theta\left(1-u^2\right)\frac{\mathrm{d}u}{\mathrm{d}\theta}.\mathrm{d}\theta \Rightarrow -\int 1-u^2\mathrm{d}u \Rightarrow -\left[u-\frac{u^3}{3}\right]$

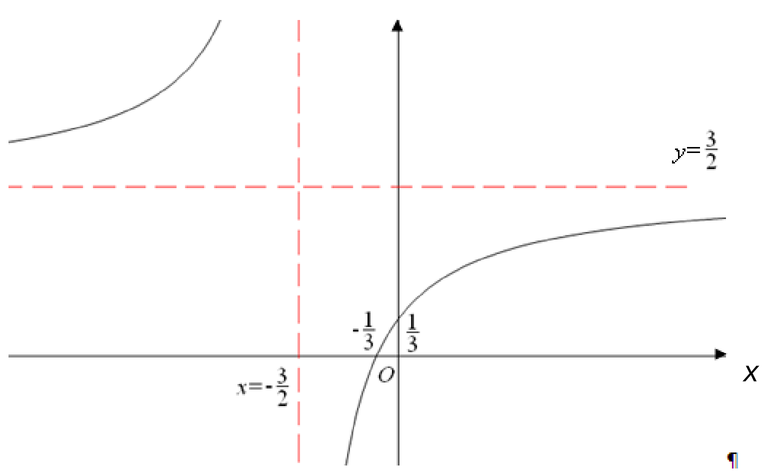
For definition of an attempt, see General Guidance

A1 for substituting $u = \cos\theta$ to give $-\cos A + \frac{1}{3}\cos^3 A(+c)$

(ii)

M1dd for substituting $\frac{\pi}{4}$ **and** 0 into their integrated expression **and** attempting to evaluate (there must be a final answer given for this mark).

A1 for the final answer as shown

Question Number	Scheme	Marks
9 (a)	(i) $y=\frac{3}{2}$	B1
	(ii) $x=-\frac{3}{2}$	B1 (2)
(b)	(i) $x=-\frac{1}{3}$ or $\left(-\frac{1}{3},0\right)$	B1
	(ii) $y=\frac{1}{3}$ or $\left(0,\frac{1}{3}\right)$	B1 (2)
(c)		B1 shape B1 asymptotes B1 crossing points (3)
(d)	$\frac{dy}{dx}=\frac{3(2x+3)-2(3x+1)}{(2x+3)^2}$	M1A1
	$x=-\frac{1}{3}$ $\frac{dy}{dx}=\frac{3\times\frac{7}{3}-2\times 0}{\left(\frac{7}{3}\right)^2}=\frac{9}{7}$	M1d
	Grad *l* = $-\frac{7}{9}$	A1
	Eqn*l*: $y=-\frac{7}{9}\left(x+\frac{1}{3}\right)$	A1ft (5)

(e)	$y=-\frac{7}{9}\left(x+\frac{1}{3}\right)=\frac{3x+1}{2x+3}$ $81x+27=-42x^2-77x-21$ $42x^2+158x+48=0$ $(3x+1)(7x+24)=0$ (or use formula) solve linear eqn ALT: $-\frac{7}{9}\times\frac{1}{3}=\frac{1}{2x+3}$ $-14x-21=27$ solve linear eqn x-coordinate of B is $-\frac{24}{7}$ $\left(\text{Accept } x=-\frac{24}{7}\right)$ correct answer	M1 M1 A1 M1 A1 (5) **[17]**

Notes

(a) (i) NOTE: If answers are transposed, award B0B1
If equations are on the graph, they must be written as equations.

B1 answer as shown. $y=\frac{3}{2}$

(ii)

B1 answer as shown $x=-\frac{3}{2}$

(b) (i)

B1 answer as shown $x=-\frac{1}{3}$ or $\left(-\frac{1}{3},0\right)$

(ii)

B1 answer as shown $y=\frac{1}{3}$ or $\left(0,\frac{1}{3}\right)$

(c)

B1 for a rectangular hyperbola with one with branches in the correct quadrants. Please be generous on the shape of the curves.

B1 for the correct asymptotes f t their answers to part (a) There must be at least one branch of their graph for the award of this mark.

B1 for the correct intersections, ft their answers to part (b)

(d)

M1 for attempting to differentiate $y=\frac{3x+1}{2x+3}$. When using quotient rule, there must be an attempt to differentiate and **subtract** the terms in the numerator; the denominator must be **squared**.

ALT (using product rule)

M1 for attempting to differentiate and add two terms.

$$\frac{dy}{dx} = 3(2x+3)^{-1} + (2x+3)^{-2}(-1)(3x+1)(2) \Rightarrow \frac{3}{2x+3} - \frac{2(3x+1)}{(2x+3)^2}$$

A1 for a fully correct differentiated expression

M1d for substituting $x = -\frac{1}{3}$ [ft their value from part (b) (i)] into their differentiated expression

Check the differentiation as they can achieve $\frac{9}{7}$ from incorrect calculus.

A1 for gradient of normal $m = -\frac{7}{9}$

A1ft uses $y - y_1 = m(x - x_1)$ to achieve an equation for the normal, where *m* is $-\frac{1}{f'(1/3)}$.

(e)

M1 for equating their straight line in the form $y = -\frac{7}{9}\left(x + \frac{1}{3}\right)$ with the equation of the curve. Simplification is not required for this mark.

M1 for simplifying their equation to form a 3TQ

A1 for the correct 3TQ

M1 for an attempt to solve their **3TQ** (please see General Guidance)

A1 for the coordinate of *B* = $-\frac{24}{7}$

ALT

M1 for equating their straight line in the form $y = -\frac{7}{9}\left(x + \frac{1}{3}\right)$ with the equation of the curve.

M1 for attempting to form a linear equation in *x* $-\frac{7}{9} \times \frac{1}{3} = \frac{1}{2x+3}$

A1 for the correct linear equation

M1 for attempting to solve their equation (moves at least one term correctly to the other side of the equality)

A1 for the coordinate of *B* = $-\frac{24}{7}$

Question Number	Scheme	Marks
10(a)	$(V=)\,50=\pi r^2 h$	B1
	$A=2\pi r^2+2\pi rh=2\pi r^2+2\pi r\times\dfrac{50}{\pi r^2}$	M1
	$A=2\pi r^2+\dfrac{100}{r}$ *	A1 (3)
(b)	$\dfrac{\mathrm{d}A}{\mathrm{d}r}=4\pi r-\dfrac{100}{r^2}$	M1
	$\dfrac{\mathrm{d}A}{\mathrm{d}r}=0\ \Rightarrow 4\pi r=\dfrac{100}{r^2}$	M1
	$r=\sqrt[3]{\dfrac{100}{4\pi}}=\sqrt[3]{7.9577...}=1.996$	A1 (3)
(c)	$\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}=4\pi+\dfrac{200}{r^3}$	M1
	$r=1.996\Rightarrow\dfrac{\mathrm{d}^2A}{\mathrm{d}r^2}>0\quad\therefore\text{min}$	M1A1 cso (3)
(d)	$A_{\text{min}}=2\pi\times1.996^2+\dfrac{100}{1.996}$	M1
	$A_{\text{min}}=75$	A1cso (2) **[11]**

Notes

(a)

B1 for equating the formula for the volume of a cylinder to the given volume of 50

M1 for substituting their $50 = \pi r^2 h$ into the formula for the surface area of a cylinder

$S = 2\pi r^2 + 2\pi rh$

A1 for the answer as shown (this is a show question; beware of 'fudging' answers)

(b)

M1 for an attempt to differentiate the **GIVEN** expression only for S

M1 for setting their $\frac{dA}{dr} = 0$

Accept $4\pi r = \frac{100}{r} \Rightarrow r = ...$

A1 for the answer as shown $r = 1.996$

(c)

M1 for attempting to find the second derivative of their $\frac{dA}{dr}$ (see General Guidance)

M1 for substituting their value of r into their $\frac{d^2A}{dr^2}$, but r must be positive.

A1 for $\frac{d^2A}{dr^2} > 0$ hence minimum cso Their value for $\frac{d^2A}{dr^2}$ must be correct (37.716.. when $r = 1.9966$.)

Also accept a conclusion by inspection. ie., 4π and $\frac{100}{r^3}$ are both positive hence $\frac{d^2A}{dr^2} > 0$

ALT

M1 for substituting a value for r < '1.996' **or** r > '1.996' to test gradient $\frac{dA}{dr}$ around r = '1.996'.

When $r < 1.996$ $\frac{dA}{dr} < 0$, when $r > 1.996$, $\frac{dA}{dr} > 0$.

M1 for substituting a value for r < '1.996' **and** r > '1.996' to test $\frac{dA}{dr}$.

A1 for conclusion; that as r increases the gradient goes from negative to positive hence, minimum.

(d)

M1 for substituting their r into the **GIVEN** expression for A and evaluating, but their r must be a positive value.

A1 for $A_{min} = 75$ cso

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