



Mark Scheme (Results)

Summer 2014

Pearson Edexcel International GCSE in
Further Pure Mathematics Paper 2
(4PM0/02)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**

- M marks: method marks
- A marks: accuracy marks. Can only be awarded if the relevant method mark(s) has (have) been gained.
- B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**

- cao – correct answer only
- ft – follow through
- isw – ignore subsequent working
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- eoo – each error or omission

- **No working**

If no working is shown then correct answers may score full marks

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

If there is a wrong answer indicated always check the working and award any marks appropriate from the mark scheme.

If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

Any case of suspected misread which does not significantly simplify the question loses two A (or B) marks on that question, but can gain all the M marks. Mark all work on follow through but enter A0 (or B0) for the first two A or B marks gained.

If working is crossed out and still legible, then it should be given any appropriate marks, as long as it has not been replaced by alternative work.

If there are multiple attempts shown, then all attempts should be marked and the highest score on a single attempt should be awarded.

- **Follow through marks**

Follow through marks which involve a single stage calculation can be awarded without working since you can check the answer yourself, but if ambiguous do not award.

Follow through marks which involve more than one stage of calculation can only be awarded on sight of the relevant working, even if it appears obvious that there is only one way you could get the answer given.

- **Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially shows that the candidate did not understand the demand of the question.

- **Linear equations**

Full marks can be gained if the solution alone is given, or otherwise unambiguously indicated in working (without contradiction elsewhere). Where the correct solution only is shown substituted, but not identified as the solution, the accuracy mark is lost but any method marks can be awarded.

- **Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded in another

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a| \text{ leading to } x = \dots$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a , b and c , leading to $x = \dots$

3. Completing the square:

$$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0 \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration:

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula:

Generally, the method mark is gained by

either quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers.

Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Question Number	Answer	Marks
1(a)	$\frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2\theta = 12, \quad \theta = \frac{2}{3}$ radians Accept 0.67 or better (If degrees formula used, M1 only when attempt to change to radians seen.)	M1,A1 (2)
(b)	arc = $r\theta = 6 \times \text{their } \theta, = 6 \times \frac{2}{3} = 4$ cm OR $12 = \frac{1}{2} \times 6 \times l, \quad l = 4$ (can be worked with degrees)	M1,A1ft (2) [4]

(a) M1 Use of a correct formula for the area of a sector.

A1 Answer correct

(b) M1 Use of a correct formula for the arc length – units for the angle used must be consistent with the formula used

A1ft Answer correct follow through their answer from (a)

Question Number	Answer	Marks
2	$\sum_{r=5}^{60} (2r + 7)$ $= \frac{56}{2}(17 + 127) \quad \text{or} \quad = \frac{56}{2}(34 + 55 \times 2)$ $= 4032$ Alternative: $\sum_{r=5}^{60} (2r + 7) = \sum_{r=1}^{60} (2r + 7) - \sum_{r=1}^4 (2r + 7)$ $= \frac{60}{2}(9 + 127) - \frac{4}{2}(9 + 15)$ $= 4032$	B1 (56) M1 (with 56 or 55 terms) A1ft (55 only) A1 [4] B1 (4) M1(4 or 5) A1ft(5 only) A1

B1 Correct number of terms seen explicitly or used

M1 Use of correct formula, either form, with 56 or 55 terms

A1ft Correct numbers, ft with 55 only

A1 Answer correct

ALT:

B1 4 terms for second sum

M1 $S_{60} - S_4$ or $S_{60} - S_5$ use of correct formula with S_4 or S_5

A1ft Correct numbers, ft 5 only A1 Correct answer

Question Number	Answer	Marks
3(a)	$\vec{OB} = 3\mathbf{i} - 4\mathbf{j} + \mathbf{i} + 7\mathbf{j} = 4\mathbf{i} + 3\mathbf{j}$ $ \vec{OB} = \sqrt{4^2 + 3^2} (=5) \quad \vec{OA} = \sqrt{3^2 + (\pm 4)^2} (=5)$ $\therefore \vec{OB} = \vec{OA} \text{ and triangle is isos.}$	B1 (can be awarded in (b)) M1 either A1 both A1cso (4)
(b)	$\pm \frac{1}{5}(4\mathbf{i} + 3\mathbf{j}) \text{ oe}$	B1ft (1) [5]

(a)

B1 A correct vector \vec{OB} seen in either (a) or (b)M1 Use of Pythagoras to find the length of OA or OB (**not** AB).

Two lengths added but final answer need not be correct

A1 Correct numbers in formulae for both lengths

A1cso Conclusion \therefore triangle is isos is sufficient

(b)

B1ft A correct unit vector, ft their \vec{OB} and $|\vec{OB}|$ (correct for their \vec{OB})

Question Number	Answer	Marks
4(a)	$4x - 4 = x^2 - 3x + 6$	M1
	$x^2 - 7x + 10 = 0$	A1
	$(x - 2)(x - 5) = 0$	M1
	$x = 2, y = 4$	A1
	$x = 5, y = 16$	A1 (5)
(b)	$x^2 - 3x + 6 \geq 4x - 4$	
	$x \leq 2 \quad 5 \leq x$	M1A1 (2) [7]

(a)

M1 Equating the line and the curve

A1 Simplified to a correct 3 term quadratic, terms in any order

M1 Solving the quadratic (see gen principles), completing to (at least one) value for x A1 Either value of x and the corresponding value of y , coordinate brackets not required.A1 The second value of x and the corresponding value of y , coordinate brackets not required.

(b)

M1 Using their x values found in (a) (or solve again) to set up inequalities. Allow with $<$, $>$

A1 also Correct inequalities.

NB: (a) Possible to substitute for x instead.

M1 Substitution

A1 Correct 3 term quadratic in x

M1 Solving

A1 A1 Correct answers, as above

Question Number	Answer	Marks
5		
(a)	$x^5 = 243, x = 3$	M1,A1 (2)
(b)	$2y + 4 = 6^2$	M1
	$y = 16$	A1 (2)
(c)	$\log_4 p + \frac{\log_4 64}{\log_4 p} = 4$ or change to base p	M1
	$(\log_4 p)^2 + 3\log_4 4 = 4\log_4 p$ or $1 + 3(\log_p 4)^2 = 4\log_p 4$	M1
	$(\log_4 p)^2 - 4\log_4 p + 3 = 0$	
	$(\log_4 p - 3)(\log_4 p - 1) = 0$	M1
	$\log_4 p = 3 \quad p = 64$	A1
	$\log_4 p = 1 \quad p = 4$	A1 (5) [9]

(a)

M1 "Undoing" the log correctly

A1 Answer correct

(b)

M1 "Undoing" the log correctly and solving the linear equation

A1 Answer correct

(c)

M1 Changing the base of either log or both changed to base 10

M1 Obtaining a 3 term quadratic in either $\log_4 p$ or $\log_p 4$ or $\log_{10} p$

M1 Solving their quadratic (see gen principles)

A1 Either value of p correctA1 The second value of p correct

Question Number	Answer	Marks
6		
(a)	$\frac{a}{1-r} = 200 \quad \frac{a(1-r^3)}{1-r} = 175 \quad \text{or} \quad a + ar + ar^2 = 175$ $200(1-r^3) = 175$ $r^3 = \frac{25}{200} = \frac{1}{8} \quad r = \frac{1}{2}$	M1A1 M1 A1 (4)
(b)	$a = 200\left(1 - \frac{1}{2}\right) = 100$	B1 (1)
(c)	$\frac{S_n}{S} = \frac{a(1-r^n)}{1-r} \div \frac{a}{1-r} = \frac{255}{256}$ $1-r^n = \frac{255}{256}$ $\left(\frac{1}{2}\right)^n = \frac{1}{256}$ $n = 8 \quad (\text{Solve by logs or inspection of powers of 2})$	M1 A1 M1A1 (4) [9]

(a)

M1 Forming either equation from the given information, formula used must be correct.

A1 Both equations to be correct.

M1 Obtaining an equation in a single variable.

A1 Obtaining the value of r .

(b)

B1 $a = 100$ If this is seen in (a) and not repeated here the mark may still be awarded.

(c)

M1 Using formulae for S_n and S and $\frac{S_n}{S} = \frac{255}{256}$ to form an equation in (a and) r (no need to simplify)A1 A correct equation without a .M1 Solving their equation to obtain a value for n A1cso $n = 8$ If $\left(\frac{1}{2}\right)^n = \frac{1}{256}$ is seen followed by $n = 8$ without any intermediate working, award M1A1Must have used $r = \frac{1}{2}$

Question Number	Answer	Marks
7		
(a)	$x^2 = 8x - 16$ or $y^2 = 8y - 16$	M1
	$x^2 - 8x + 16 = 0$ or $y^2 - 8y + 16 = 0$	
	$(x - 4)^2 = 0$ $x = 4, y = 4$	M1A1,A1 (4)
(b)	$\int_2^4 \pi y^2 dx = \pi \int_2^4 (8x - 16) dx$	M1
	$= \pi [4x^2 - 16x]_2^4$	DM1
	$= \pi [64 - 64 - (16 - 32)] = 16\pi$	A1
	Vol of cone $= \frac{1}{3} \pi \times 4^2 \times 4 = \frac{64}{3} \pi$ (or by integration using $y = x$)	B1
	Required vol $= \frac{64}{3} \pi - 16\pi = \frac{16}{3} \pi$	B1ft (5) [9]

(a)

M1 Eliminating y or x between the line and the curve equation.M1 Solving their resulting quadratic to obtain a value for either x or y A1 x or y correct

A1 other variable correct

ALT: Using $y = x$ as a tangentM1 Differentiate $y^2 = 8x - 16$ square root and chain rule or implicitM1 Substitute $\frac{dy}{dx} = 1$ and solve for x or y

A1, A1 Correct answers as above

(b)

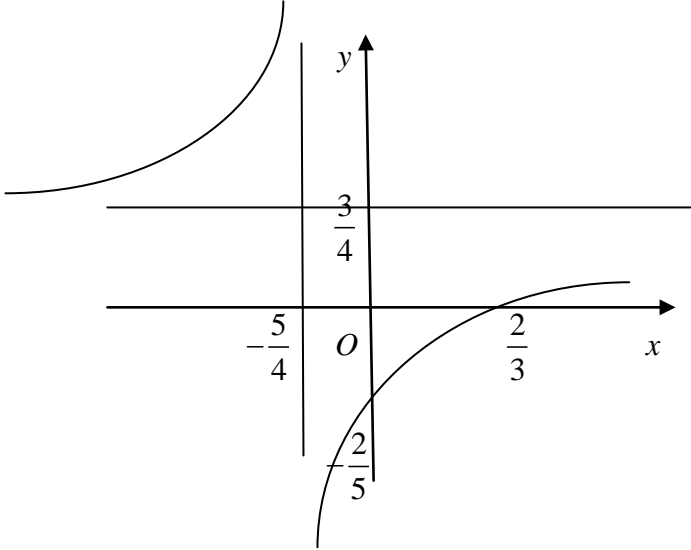
M1 Using $\int_2^4 \pi y^2 dx$ for the curve. Limits not needed.

DM1 Attempting the integration. Ignore limits.

A1 Obtaining 16π , using correct limitsB1 Obtaining volume of cone $\frac{64}{3} \pi$

B1ft Subtracting their vol. of rev. from their vol of cone. Award only if answer is positive.

Use of $\pi \int_2^4 \{x^2 - 8(x - 2)\} dx$. as above A1for $\frac{8\pi}{3}$ B1 for cone height $2 \left(\frac{8\pi}{3} \right)$ B1ft adding the volumesUse of $\pi \int_2^4 \{x - 8(x - 2)\} dx$ scores M0 (dimensionally incorrect) but B1B1 may be available

Question Number	Answer	Marks
8		
(a)	(i) $y = \frac{3}{4}$ (ii) $x = -\frac{5}{4}$	B1B1 (2)
(b)	(i) $\left(\frac{2}{3}, 0\right)$ (ii) $\left(0, -\frac{2}{5}\right)$	B1B1 (2)
(c)		B1 (2 branches) B1ft (asymptotes) B1ft (crossing points) (3)
(d)	$\frac{dy}{dx} = \frac{3(4x+5) - 4(3x-2)}{(4x+5)^2}$ $x = -1 \quad \frac{dy}{dx} = \frac{3 - 4 \times -5}{1} = 23$ $x = -1 \quad y = \frac{-5}{1} = -5$ $y + 5 = -\frac{1}{23}(x + 1)$ $x + 23y + 116 = 0$	M1A1 A1 B1 M1A1ft A1 (7) [14]

(a)

(i) B1 $y = \frac{3}{4}$ oe (ii) B1 $x = -\frac{5}{4}$ oe **must** be equations

If correct equations seen but labelled the wrong way round, give B0B1

(b)

(i) B1 (i) $\left(\frac{2}{3}, 0\right)$ or $x = \frac{2}{3}$ (ii) B1 $\left(0, -\frac{2}{5}\right)$ or $y = -\frac{2}{5}$

If correct coordinates seen but labelled the wrong way round, give B0B1

(c)

B1 2 branches in the correct "quadrants"

B1ft Asymptotes shown with at least one branch of the curve. At least one branch must be asymptotic to each line. Neither branch of the curve must meet or cross either asymptote.

B1ft Coordinates of the points where the **curve** crosses the axes shown. (0s not needed). There must be no extra crossing points

Follow through only if the answers used are consistent with 2 branches in the correct quadrants (first B1 need not be earned)

(d)

M1 Quotient rule used. Denominator must be the square of the original denominator. The numerator must be the difference of two terms of the appropriate form (but may be the wrong way round). Alt, use product rule, A1 Fully correct differential.

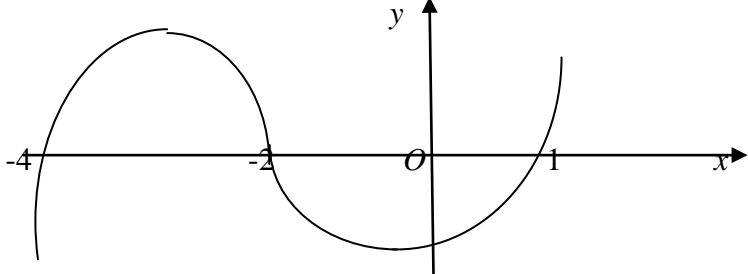
A1 Using $x = -1$ to obtain $\frac{dy}{dx} = 23$

B1 Using $x = -1$ to obtain $y = -5$

M1 Using any correct, complete method to obtain an equation of the normal. The gradient must be $\frac{-1}{\text{their } \frac{dy}{dx}}$

(not dependent on first M mark)

A1ft Correct numbers used, follow through their $\frac{dy}{dx}$ and y A1cso $x + 23y + 116 = 0$ or any integer multiple of this Accept in any order on lhs provided rhs is 0

Question Number	Answer	Marks
9(a)	$x = -2 \quad -8 + 20 - 2p - q = 0$	M1
	$x = 1 \quad 1 + 5 + p - q = 0$	A1 (2)
(b)	$2p + q = 12$ $p - q = -6$	
	$3p = 6 \quad p = 2 \quad *$	M1A1cso
	$q = 8$	B1 (3)
(c)	$f(x) = (x+2)(x-1)(x+4)$	B1 (1)
(d)		B1 (shape) B1 (x coords) (2)
(e)	$\text{Area} = \int_{-\frac{4}{3}}^2 (x^3 + 5x^2 + 2x - 8 - (x^3 + 2x^2 + 4x)) dx$ <p>(curves either way round for full marks if earned. Final answer must be positive)</p> $= \int_{-\frac{4}{3}}^2 (3x^2 - 2x - 8) dx$ $= \left[x^3 - x^2 - 8x \right]_{-\frac{4}{3}}^2$ $= 8 - 4 - 16 - \left(\left(-\frac{4}{3} \right)^3 - \left(-\frac{4}{3} \right)^2 - 8 \times \left(-\frac{4}{3} \right) \right)$ $= -18.5 \quad \text{Area must be positive } \therefore \text{area} = 18.5$	M1 M1A1 DM1 A1 (5) [13]

(a)
M1 Using either given factor and the factor theorem to form an (unsimplified) equation in p and q
A1 Obtaining 2 correct (unsimplified) equations in p and q

(b)
M1 Solving their equations to obtain a value for p or q

A1cso for $p = 2$ *

A1 for $q = 8$ (This can be awarded as a B mark if $p = 2$ used and equations not solved.)

(c)
B1 Product of 3 correct linear factors

(d)
B1 Sketch showing a cubic graph, **crossing** the x -axis at 3 (correct) points and the correct way up
B1 Correct x coordinates written on the sketch or shown by graduated markings

(e)
M1 Using the integral of the difference of the 2 equations (either way round). Limits not needed here
M1 Integrating their expression (see gen principles); can be done with an unsimplified form. Limits not needed

A1 Correct integration and correct limits seen

DM1 Substituting the correct limits Dependent on both M marks

A1 Obtaining Area = 18.5 Negative answer scores A0 **Must** be 3 significant figures.

May be done by 2 completely separate integrals. If only one function is used no marks.

Question Number	Answer	Marks
10		
(a)	(i) $\cos 2A = \cos^2 A - \sin^2 A, = (1 - \sin^2 A) - \sin^2 A$ $= 1 - 2\sin^2 A$ *	M1,M1 A1cso
	(ii) $\sin 2A = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A$	B1 (4)
(b)	$\sin 3A = \sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A$ $= 2 \sin A \cos^2 A + (1 - 2 \sin^2 A) \sin A$ $= 2 \sin A (1 - \sin^2 A) + (1 - 2 \sin^2 A) \sin A$ $= 3 \sin A - 4 \sin^3 A$ *	M1 M1 M1 A1 cso (4)
(c)	$-4(3 \sin x - 4 \sin^3 x) + 1 = 0$ $4 \sin 3x = 1$ $3x = 0.2526, 2.888, 6.535, 9.172$ $x = 0.0842, 0.963, 2.18, 3.06$	M1 M1 A1A1 (4)
(d)	$\int (24 \sin^3 \theta + 6 \cos \theta) d\theta = \int (18 \sin \theta - 6 \sin 3\theta + 6 \cos \theta) d\theta$ $= [-18 \cos \theta + 2 \cos 3\theta + 6 \sin \theta] (+c)$	M1 A1 (2)
ALT:	$\int (24 \sin^3 \theta + 6 \cos \theta) d\theta = 6 \int \{4(1 - \cos^2 \theta) \sin \theta + \cos \theta\} d\theta$ $= [-24 \cos \theta + 8 \cos^3 \theta + 6 \sin \theta] (+c)$	
(e)	$= [-18 \cos \theta + 2 \cos 3\theta + 6 \sin \theta]_0^{\frac{\pi}{3}} = -9 - 2 + 3\sqrt{3} - (-18 + 2)$ $= 5 + 3\sqrt{3}$ or $= 5 + \sqrt{27}$	DM1 A1 (2) [16]

(a)

(i) M1 Making $A = B$ in the cos identity

M1 Using the Pythagorean identity to eliminate $\cos^2 A$

A1cso $\cos 2A = 1 - 2\sin^2 A$ *

(ii) B1 A correct expression for $\sin 2A$. No working or simplification needed

(b)

M1 Using $\sin(A + B)$ to obtain an expression with $\sin/\cos 2A/A$.

M1 Using the results in (a) to obtain an expression in $\sin A$ and $\cos A$

M1 Using the Pythagorean identity to eliminate $\cos^2 A$

A1cso Obtaining $3\sin A - 4\sin^3 A$ *

If (b) is worked in reverse, the 3 M marks must be given exactly as above and the A mark given if everything is correct

(c)

M1 Using the result from (b) to obtain an equation with $\sin 3x$ and no other trig function

M1 Obtaining any correct value for $3x$ Can be in degrees or radians

A1A1 Correct answers in radians. Any 2 or 3 correct scores A1. All 4 correct scores A1A1.

Rounding: See General Principles

(d)

M1 Using the result from (b) or the Pythagorean identity to obtain an expression which can be integrated and attempting the integration

A1 Correct integration constant not required

(e)

DM1 Substituting the given limits in *their* result in (d) including obtaining numerical values for the trig functions. Dependant on the M mark in (d)

A1cso obtaining $5 + 3\sqrt{3}$ or $5 + \sqrt{27}$ Both marks in (d) needed for this.

If this work is not labelled (e), instead just being a continuation of (d), the marks for (e) can still be awarded

Question Number	Answer	Marks
11		
(a)	$4 \times 7x = (3x-1)^2 + (3x+1)^2 - 2(3x-1)(3x+1)\cos 60^\circ$ $28x = 9x^2 - 6x + 1 + 9x^2 + 6x + 1 - (9x^2 - 1)$ $9x^2 - 28x + 3 = 0$ $(9x-1)(x-3) = 0 \quad *$	M1A1 A1 (3)
(b)	$\therefore x = 3$ $x \neq \frac{1}{9} \text{ as } AB \text{ would be negative alt; } x > \frac{1}{3} \text{ as } AB \text{ must be positive}$	B1 B1 (2)
(c)	$\frac{\sin B}{10} = \frac{\sin 60}{2\sqrt{21}}$ $\sin B = \frac{10\sin 60}{2\sqrt{21}} \quad B = 70.89\dots = 70.9^\circ$	M1A1ft A1 (3)
(d)	$\text{Area} = \frac{1}{2} AB \times AC \sin 60$ $= \frac{1}{2} \times 8 \times 10 \times \frac{\sqrt{3}}{2} = 20\sqrt{3}$	M1 A1 (2)
		[10]

(a)

M1 Using the cosine rule. The formula must be correct, either shown explicitly or implied by the substitution shown

A1 All terms correct in the formula Simplification of terms not needed.

A1 cso Correct result NB Given answer, check all working

(b)

B1 $x = 3$

B1 Giving a suitable reason for $x \neq \frac{1}{9}$

If both values are given and $\frac{1}{9}$ is not ruled out, give B0B0; if only $x = 3$ is seen w/o a reason, award B1B0

(c)

M1 Using the sine or cosine rule

A1ft Correct numbers in the formula, follow through their value of x

A1cao $B = 70.9^\circ$ **must** be 1 decimal place

(d)

M1 Using any correct, complete method to obtain a value for the area of $\triangle ABC$. The value need not be exact or even simplified

A1 Obtaining $20\sqrt{3}$ or any **exact** equivalent of that. Ignore decimal answers following a correct exact answer.

