Write your name here

Surname	Other names

Edexcel
International GCSE

Centre Number | | | | | |

Candidate Number | | | | |

Further Pure Mathematics

Paper 1

Thursday 19 January 2012 – Morning
Time: 2 hours

Paper Reference
4PM0/01

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P40616A

6/6/6/6/6/4/2/2

P 4 0 6 1 6 A 0 1 3 2

PEARSON

Answer all ELEVEN questions

Write your answers in the spaces provided

You must write down all stages in your working

1 Show that the two lines with equations

$$6x + 4y = -15$$

$$10x - 15y = 9$$

are perpendicular.

(4)

(Total for Question 1 is 4 marks)

2 Solve the equation

$$\frac{x}{x+1} - \frac{1}{x+2} = 2$$

Give your answers correct to 3 significant figures.

(4)

(Total for Question 2 is 4 marks)

3 Solve the inequality $6x^2 - 19x - 7 < 0$

(4)

(Total for Question 3 is 4 marks)

4 Find the coefficient of x^7 in the expansion of $\left(1+\frac{x}{\sqrt{3}}\right)^{10}$, giving your answer in the form $a\sqrt{3}$, where a is a rational number.

(4)

(Total for Question 4 is 4 marks)

5 Differentiate with respect to x

(a) $y = x^2 e^x$

(2)

(b) $y = (x^3 + 2x^2 + 3)^5$

(3)

Question 5 continued

(Total for Question 5 is 5 marks)

6 For x radians,

$$y = 3\cos\frac{x}{2}$$

(a) Complete the table, giving the three missing values correct to 2 decimal places.

(2)

x	0	0.5	1	1.5	2	2.5	3	3.5
y	3		2.63	2.20		0.95	0.21	

On the axes opposite,

(b) draw the graph of $y = 3\cos\frac{x}{2}$ for $0 \leqslant x \leqslant 3.5$

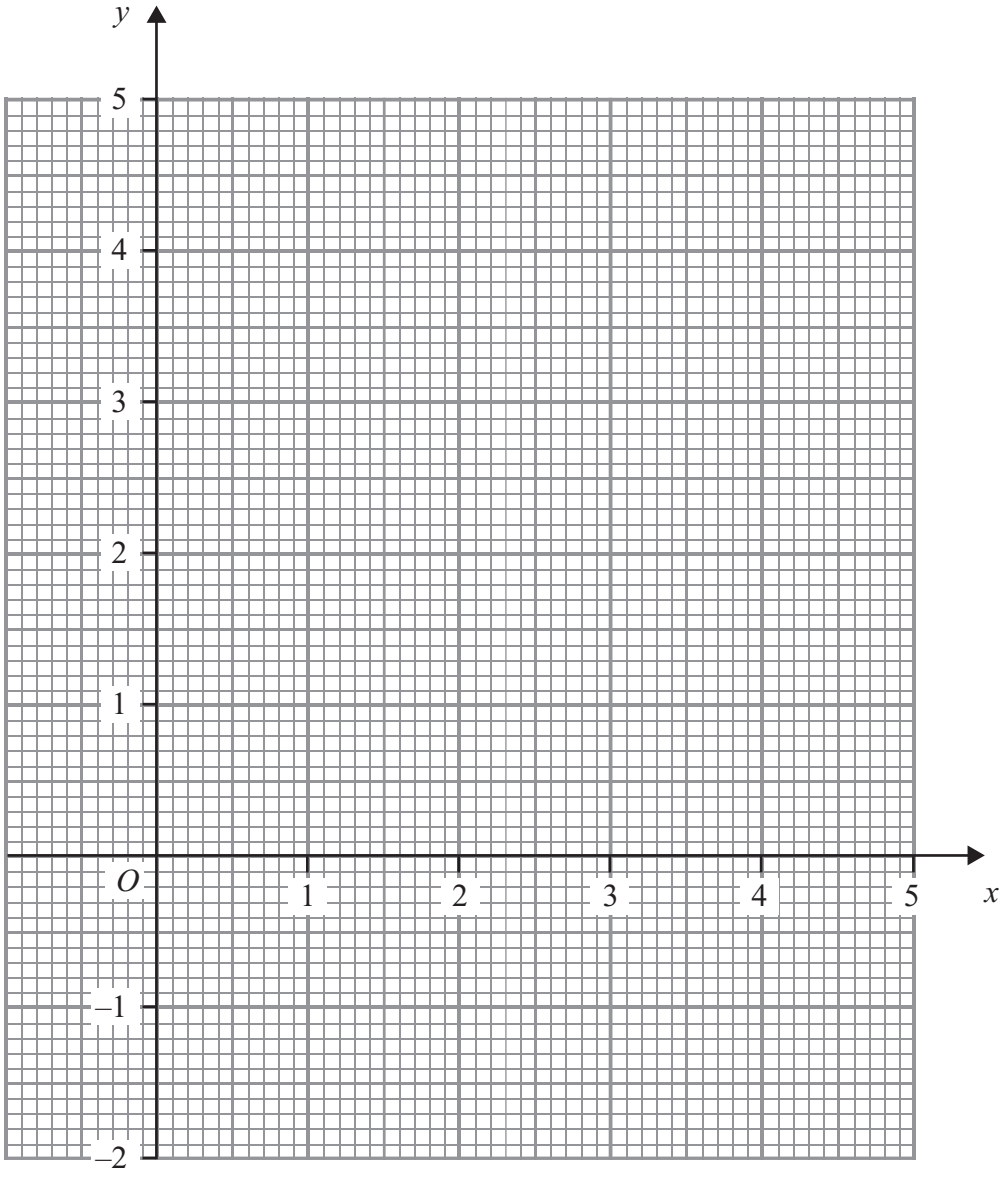
(2)

(c) Using your graph, find an estimate, to 1 decimal place, for the root of the equation

$$2x = 1 + 2\cos\frac{x}{2}$$

(4)

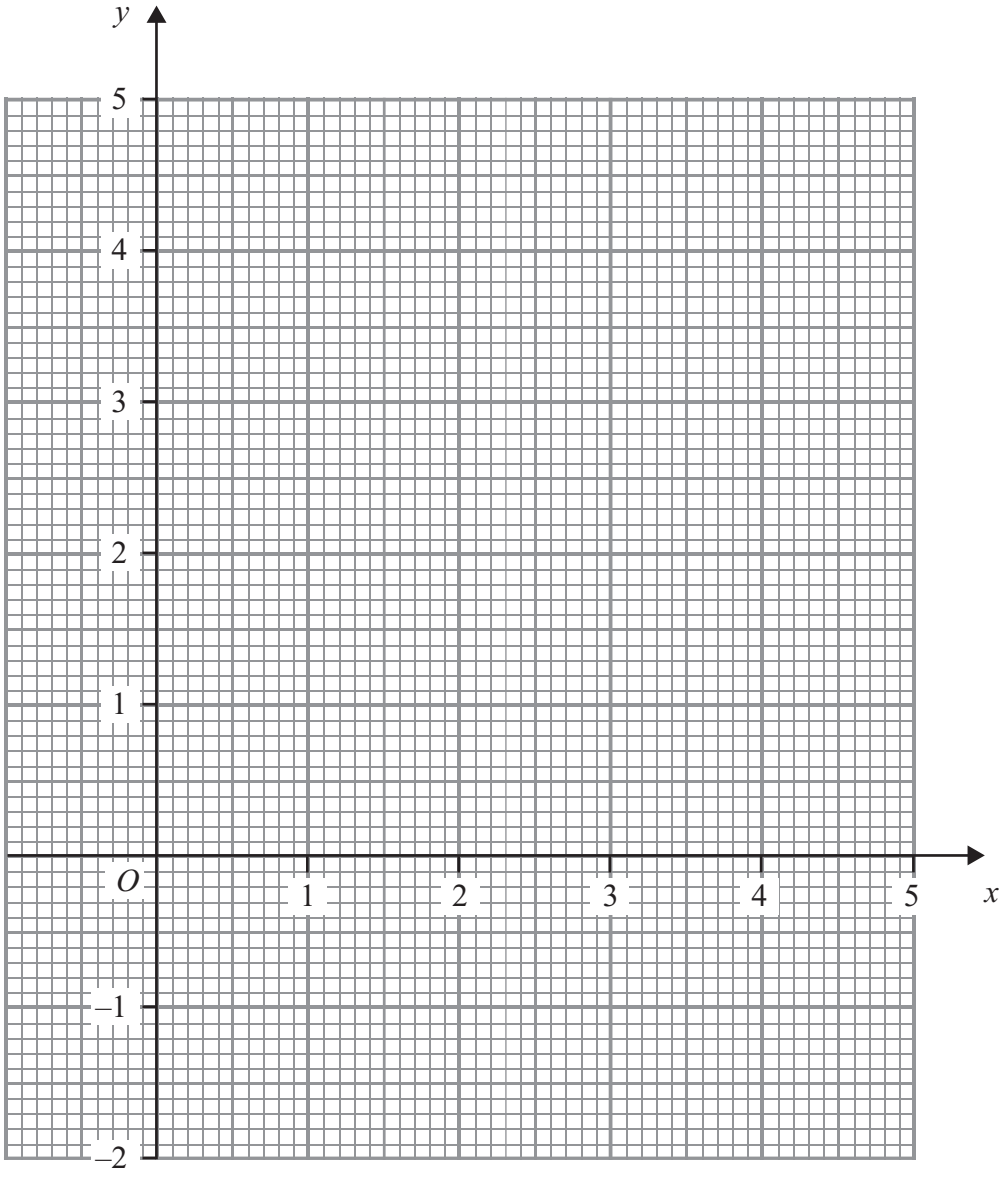
Question 6 continued



Question 6 continued

Question 6 continued

Use this page only if you need to redraw your graph.



(Total for Question 6 is 8 marks)

7 The curve C with equation $y = \dfrac{2x-3}{x-3}$, $x \neq 3$, crosses the x-axis at the point A and the y-axis at the point B.

(a) Find the coordinates of A and the coordinates of B.

(2)

(b) Write down an equation of the asymptote to C which is

(i) parallel to the y-axis,

(ii) parallel to the x-axis.

(2)

(c) Sketch C showing clearly the asymptotes and the coordinates of the points A and B.

(3)

(d) Find an equation of the normal to C at the point B.

(5)

The normal to C at the point B crosses the curve again at the point D.

(e) Find the x-coordinate of D.

(4)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 16 marks)

8 The equation $x^2 + mx + 15 = 0$ has roots α and β and the equation $x^2 + hx + k = 0$ has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(a) Write down the value of k

(1)

(b) Find an expression for h in terms of m

(6)

Given that $\beta = 2\alpha + 1$

(c) find the two possible values of α

(3)

(d) Hence find the two possible values of m

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 13 marks)

9

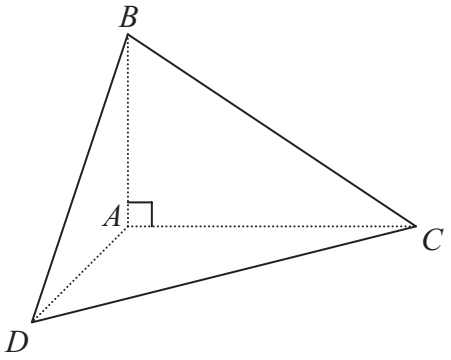


Figure 1

Figure 1 shows a triangular pyramid $ABCD$.

$\angle BAC = \angle DAC = \angle BAD = 90°$

$AD = 5$ cm, $AC = 8$ cm and $AB = 6$ cm.

(a) Find, in degrees to the nearest 0.1°, the size of $\angle BDC$.

(6)

(b) Find, to 3 significant figures, the area of triangle BDC.

(3)

(c) Find the area of triangle DAC.

(1)

The point E lies on CD so that AE is perpendicular to CD.

(d) Find the exact length of AE.

(2)

(e) Hence, or otherwise, find in degrees to the nearest 0.1°, the size of the angle between the planes DAC and BDC.

(4)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 16 marks)

10

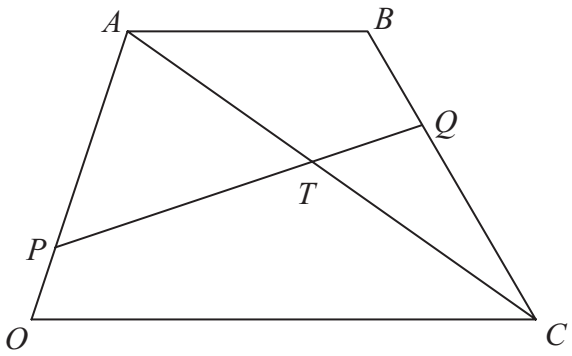


Figure 2

Figure 2 shows a trapezium $OABC$ in which AB is parallel to OC and $AB = \frac{1}{2} OC$. The point P divides OA in the ratio 1:3 and the point Q divides BC in the ratio 1:2

The line AC intersects the line PQ at the point T.

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$

(a) Find, as simplified expressions in terms of **a** and **c**

(i) $\overrightarrow{BC}$

(ii) $\overrightarrow{PQ}$

(5)

(b) (i) Given that $\overrightarrow{PT} = \lambda \overrightarrow{PQ}$, find an expression for $\overrightarrow{AT}$ in terms of λ, **a** and **c**

(ii) Given also that $\overrightarrow{AT} = \mu \overrightarrow{AC}$, find an expression for $\overrightarrow{AT}$ in terms of μ, **a** and **c**

(2)

(c) Use your answers from part (b) to find the value of λ and hence write down the ratio $PT : TQ$

(6)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 13 marks)

11

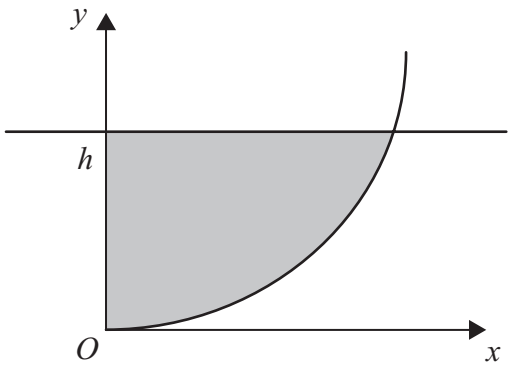


Figure 3

The centre of the circle C, with equation $x^2 + y^2 - 10y = 0$, has coordinates (0, 5). The circle passes through the origin O. The region bounded by the circle, the positive y-axis and the line $y = h$, where $h < 5$, is shown shaded in Figure 3. The shaded region is rotated through 2π radians about the y-axis.

(a) Show that the volume of the solid formed is $\frac{1}{3}\pi h^2(15 - h)$.

(5)

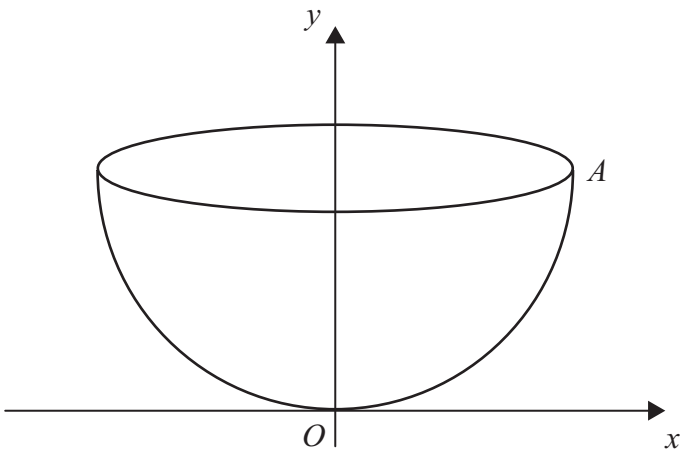


Figure 4

The point A with coordinates (5, 5) lies on C. A bowl is formed by rotating the arc OA through 2π radians about the y-axis, as shown in Figure 4. Water is poured into the bowl at a constant rate of 6 cm^3/s. The volume of water in the bowl is V cm^3 when the depth of water above O is h cm.

(b) Use the formula given in part (a) to find an expression for $\frac{dV}{dh}$ in terms of h.

(1)

(c) Find, to 3 significant figures, the rate at which h is changing when the water above O is 1.5 cm deep.

(4)

The area of the surface of the water is W cm^2 when the depth of water above O is h cm.

(d) Show that, for $0 < h < 5$, the rate of change of the depth of water above O is $\frac{k}{W}$, stating the value of k.

(3)

Question 11 continued

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS